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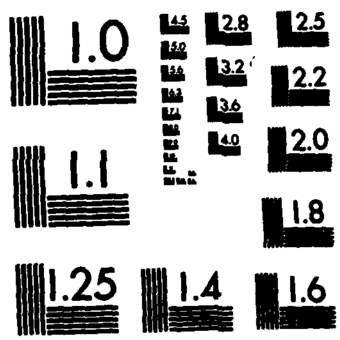

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A Statistical Analysis of the Distribution of D062 Demand in a Given Leadtime

by

W. Steven Demmy

September 1981

WP-81-03
Decision Systems
2125 Crystal Marie Drive
Beavercreek, Ohio 45431
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper presents the results of several statistical studies of the distribution of forecast errors associated with the forecasting rules currently used in the Economic Order Quantity (EOQ) Buy Computation System (D062). Actual demand histories for over 20,000 Air Force EOQ items for the CY71-79 interval served as the major data source used in the study. Useful analytic approximation formulas are presented in Sections III and IV.

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I-1

Section I

Introduction

Background

An important characteristic of any inventory control system is the probability distribution of forecast errors. Such a distribution quantifies the percent of time that forecast demand is above or below the forecasted values, and the magnitude and frequency of these errors. Such information is critical in constructing cost effective safety levels for individual inventory items. At present, the safety level calculations utilized in the Economic Order Quantity (EOQ) Buy Computation System (D062) assume that errors in forecasting the total number of units demanded in a procurement leadtime are normally distributed. In testing this assumption, however, Demmy (1979) observed that aggregate forecast errors were described by a highly skewed distribution. One explanation for this result is that the demand process itself is best described by a stationary skewed distribution. For example, the negative binomial and several of the compound Poisson processes have this characteristic. On the other hand, Hayya (1980) has suggested that the observed error distribution may be the result of combining the biased forecast errors that result when moving average, "straight-line" projections are used to forecast a mixture of items that are in fact subject to significant up and down trends. Still another explanation is that the demand process is described by a non-stationary, skewed distribution. There may of course be many other explanations for the observed aggregate error distributions.

In this paper, we investigate the characteristics of forecast errors associated with current D062 forecasting methods. Our goal is to obtain useful descriptions for the distribution of demands for EOQ items that will be convenient for use in safety level computations. The paper consists of several major sections. In section I, we describe the current forecasting methods utilized in the D062 system in more detail, and we review the current assumptions embedded in D062 safety level computations. In Section II, we describe the results of several preliminary investigations into the nature of EOQ demand forecast errors. This section presents plots of ten year demand histories for representative EOQ items, and also presents the results of preliminary studies of the consistancy of forecast errors across aircraft, time periods, and demand rate classes. In Section III, we present refined models of the distribution of EOQ demand forecasting errors and we present useful approximation formulas. Finally, Section IV summarizes the major findings of this report.

Large amounts of data were analyzed during this study. To reduce the volume of this specific report, a majority of the background data is presented in separately bound Appendices.

Let us now discuss the characteristics of current D062 methods for forecasting EOQ demands.

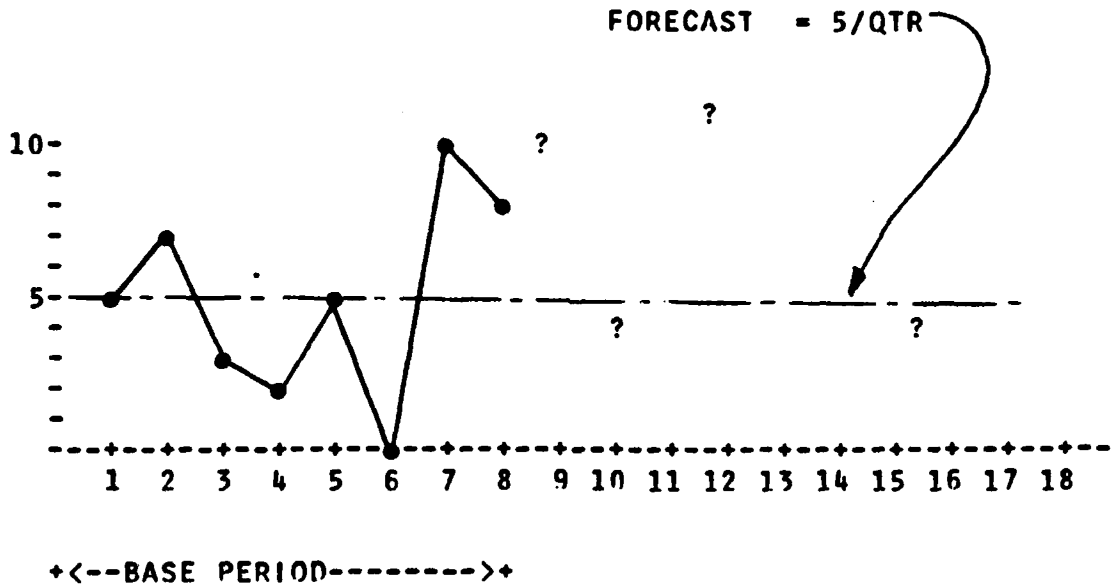
Current D062 Forecast Methods

AFLCR 57-6 contains a detailed description of AFLC EOQ demand forecasting methods. In this section, we briefly outline these procedures. Basically, the current D062 forecast for recurring EOQ demands is based upon a eight quarter moving average, with proportional adjustments for projected changes in associated aircraft flying programs. For example, Figure I-1 illustrates an item with an average demand of 5 units per quarter during the past 8 quarters. Suppose future flying activity is forecasted to remain at the same rate as was observed during the past 8 quarters. In this case, D062 would forecast 5 units of demand per quarter for each future quarter, as illustrated by dashed line in Figure I-1. On the other hand, if future flying activity was expected to be 20 percent higher in the next 8 quarters than that experienced during the previous 8 quarters, the forecast quarterly demand would be (1.20) x (5) = 6 units per quarter.

Figure I-1 also illustrates the calculation of a MAD for this hypothetical item. MAD stands for Mean Absolute Deviation, a measure of the average difference between the 8 quarter average and each of the 8 data values that were actually observed. If no change in flying program activity is expected, current forecast rules then use this historical MAD as an estimate of the MAD of demand in each future quarter. On the other hand, if future flying activity is expected to be either higher or lower than that experienced in the past two years, the MAD associated with future quarterly demand is set equal to

$$\text{Future MAD} = R^{.85} \times (\text{Old MAD})$$

F O R E C A S T S



PERIOD	DEMAND	DEVIATION	ABSOLUTE DEVIATION
1	5	0	0
2	7	+2	2
3	3	-2	2
4	2	-3	3
5	5	0	0
6	0	-5	5
7	10	+5	5
8	8	+3	3
---	---	----	---
TOTAL	40	0	20
	AVE= 5/QTR	0	2.5
		BIAS	MAD

Figure I-1. Example of an Eight Quarter Average Forecast.

where R denotes the ratio of projected flying activity during the future two years to the flying activity observed during the past two years. This adjustment is based on statistical relationships observed by Robert Stevens of AFLC/XRS which indicates that the standard deviation of demand per period is proportional to mean demand activity raised to the .85 power.

The specific data values used in computing the 8 quarter moving average depend upon the Supply Management Grouping Code (SMGC) associated with a given EOQ item. T Figure I-2 illustrates the basic rules. For example, items with SMGC codes of X and P (i.e. items with projected demand rates less than $5,000 per year) utilize gross demands in both the average and MAD calculations, where gross demands are the sum of sales demands, transfer demands, Foreign Military Sales (FMS), and non-recurring demands. Serviceable returns are not included in this calculation. On the other hand, items with SMGC codes of P and M (i.e. items with annual demands greater than $5,000 per year) utilize net demand in the average and MAD calculations. In this case, sales, transfer, and FMS demands are summed and then past serviceable returns are subtracted from this total. This gives net quarterly demand for each of the past 8 quarters. Both average and MAD values are based on net demands for SMGC P and M items.

<u>Data Used in this Study</u>

To develop the distribution of forecast errors associated with the above forecasting methods we utilized historical data from several major sources. First, data describing both forecasted and observed aircraft flying programs were obtained from the K004 and G033J systems, respectively, while information on actual D062 EOQ demand histories were obtained from the INSSIM Data Bank. The D062 System maintains a weapon code

FORECAST METHOD DEPENDS UPON SMGC

SMGC	FORECAST* IS BASED ON
X and T	GROSS DEMANDS (sales + transfer + FMS + non-recurring)
P and M	NET DEMANDS (sales + transfer + FMS less serviceable returns)

*Both Mean and MAD values are computed using these rules.

Figure I-2. Forecast Calculations by Supply Management Grouping Code.

for each EOQ item. This code identifies the primary aircraft supported by the specific EOQ item. Actual aircraft flying programs associated with a given aircraft code for the period CY73-CY79 were obtained through the G033J system from output product A-G033J-PAR-M1-MMO. This information was collected for the interval July 1970 through June 1979. Predictions of flying program activity which were made for each quarter throughout the 1970s were obtained from the K004 data system, using report K004.D81A RCS: NR-LOG-LR(AR)7208. Reference 2 presents the detailed data for actual and predicted program activity collected in this process. In addition, Reference 2 presents plots of both the actual and predicted program activity, and plots of the ratio predicted to actual flying programs for each aircraft associated with the INSSIM Data Bank. We refer to this latter ratio as the "forecast accuracy ratio."

D062 Demand Data

The item data used in this study were obtained from the INSSIM Data Bank. This data bank currently contains demand histories for thousands of D062 items managed by the Sacramento, Oklahoma City and Warner Robins Air Logistics Centers (ALC) for the interval CY 71-79, a total of 38 quarters of demand data.

The INSSIM Data Bank was constructed over a period of years by AFLC/XRS by Mr. Fred Conway and Mr. Armin Rubbert. In building this data set, several problems were encountered regarding the availability of data in past years and the readability of the "old" tapes in the data bank. As a result, the following rules were adopted in building the historical data files:

1. Sales, Transfer, and FMS demands were combined to provide a single demand total per quarter.

2. Non-recurring demands were not available and, thus, were not included in the data bank records.

3. Sales returns and Transfer returns were combined to provide a single value for serviceable returns by quarter.

4. Lead time and inventory management codes were obtained from the records corresponding to the first quarter of FY75. This includes the weapon code, the code which identifies the primary aircraft associated with each EOQ item.

5. On-hand and on-order assets were obtained from the oldest available D062 stock status record. This corresponded to the first quarter of FY74.

6. Items with Special Codes of C, D, E, I, M, X, U, or N in any fiscal year were deleted from the file. Requirements for items with these codes are computed using manual methods, and consequently were not candidates for inclusion in the INSSIM Data Bank.

7. Items with incomplete demand histories were also deleted from the Data Bank. That is, an item was included in the INSSIM Data Bank only if demand history records were present in the D062 system for each of the fiscal years in the interval CY71-79.

Hence, this rule eliminates from consideration all items which either enter or leave a specific ALC's data files during the CY71-79 interval, or which were transferred from one ALC to another during the interval. Items which were switched from management under the D062 computation system to the D041 system, or vice versa, would also be eliminated by this rule.

Statistical Studies

Once the historical data files were available, a large number of statistical studies were performed with this data base. Figure I-3 illustrates the major types of analyses which were done. As illustrated in the Figure, our statistical analysis data system had four major components. First, as illustrated in Block A, we developed detailed item plots of demands and returns for several hundred specific items from the INSSIM data bank, and we then manually reviewed each of these plots in an attempt to discover underlying trends or other patterns that were shared among the items. Reference 3 presents plots of a sample of one hundred items studied in this process. As shown in Block B, a second major step involved the analysis of standardized forecasting errors Zi associated with individual period forecasts. In these studies, we attempted to evaluate the consistency of forecasting errors from time period to time period and to discover if there were any significant error differences among the distributions of period forecast errors among aircraft, time periods, demand classes, or other stratifications of D062 items. Analysis of variance (ANOVA), correlation, frequency distribution analysis, and cross tabulations were the primary analysis tools used in this step. As shown in Block C, similar analyses were performed for the distribution of errors LTZi associated with forecasts for a given number of time periods. The results of the analysis steps illustrated in Blocks A, B, and C are discussed in Section II. Based on the results of these analyses, we then sought appropriate transformations of the observed distributions of forecast errors which could be used to simplify the calculation of individual safety level policies. This is illustrated by Block D in Figure I-3. The results of these efforts are discussed in Section III. Let us now discuss the precise definitions of the statistics Zi and LTZi which were the subject of these analyses efforts.

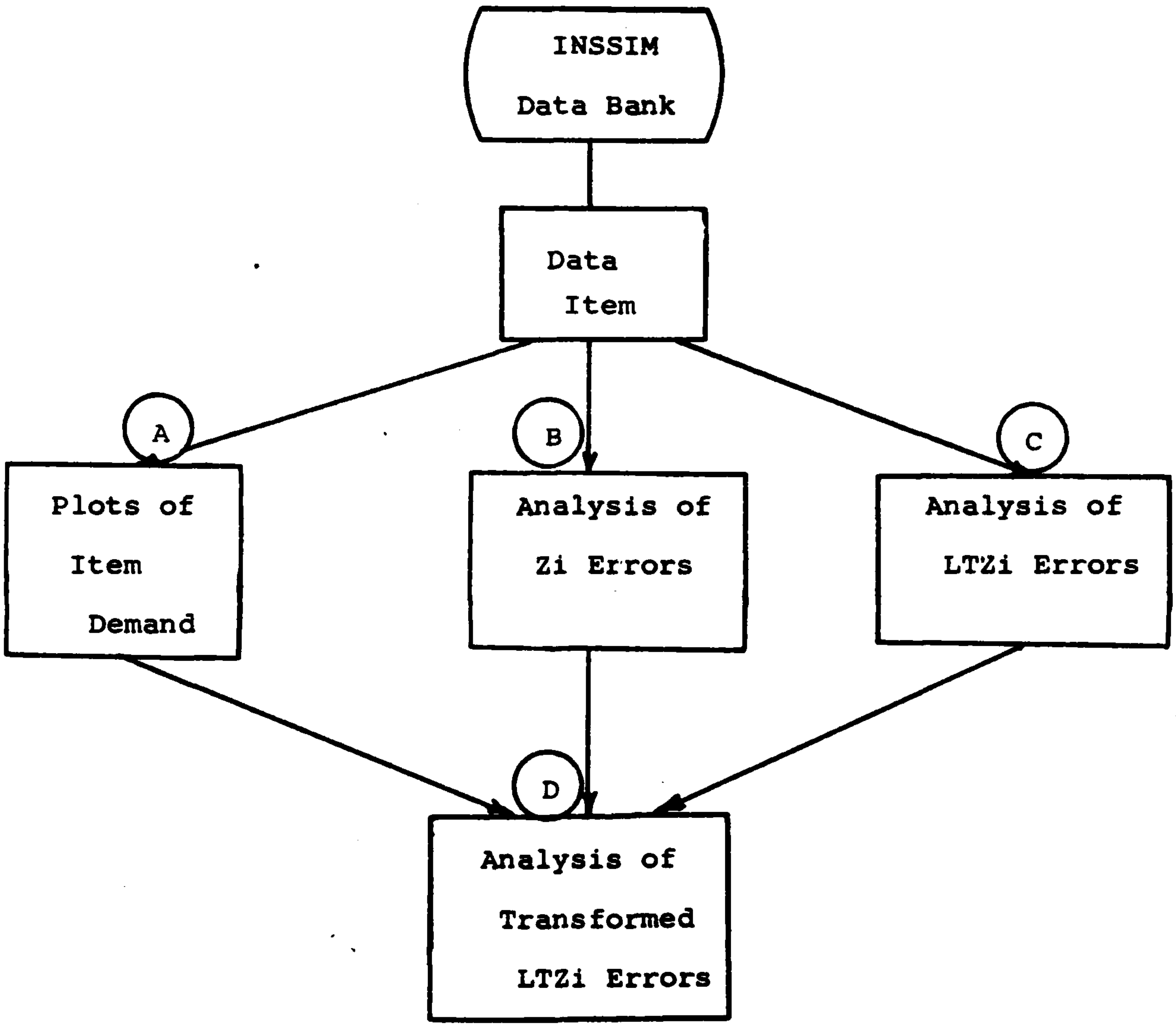


Figure I-3. Major Analysis Steps.

Definition of Standardized Errors

In performing these studies it was necessary to normalize our data so that errors associated with different items might be compared. Two types of standardized errors were computed. We refer to these as "period errors", which we denote by the symbol "Zi", and the "leadtime demand error", which we denote by the symbol "LTZi". The computation formulas for these errors are presented in Figure I-4. As shown in the figure, the standardized period error Zi associated with quarter i is computed by subtracting the forecasted demand for quarter i from the actual demand observed in this quarter. This result is then divided by the 8 quarter historical MAD. The standardized leadtime error LTZi is computed in a similar manner, but in this case total forecasted net demand for the given leadtime is subtracted from the total observed demand for this interval. The result is then divided by MAD.

Numbering Conventions

As noted above, 38 quarters of data were available for this study, covering the interval CY71-CY79. For convenience, we refer to the first quarter of CY71 as quarter 1, the next quarter as quarter 2, and so on. This numbering scheme is illustrated in Figure 5. Hence, quarter 9 refers to the first quarter CY73 and quarter 20 refers to the last quarter of CY75.

STANDARDIZED ERRORS

PERIOD ERROR

$$ZI = \frac{\left(\begin{array}{c}\text{ACTUAL NET}\\\text{DEMAND}\end{array}\right) - \text{FORECAST}}{\text{MAD}}$$

LEAD TIME DEMAND ERROR

$$LT\ ZI = \frac{\left(\begin{array}{c}\text{TOTAL NET}\\\text{DEMAND}\\\text{IN}\\\text{I PERIODS}\end{array}\right) - \left(\begin{array}{c}\text{FORECAST}\\\text{DEMMAND}\\\text{IN}\\\text{I PERIODS}\end{array}\right)}{\text{MAD}}$$

Figure I-4. Formulas for Standardized Errors.

Since our data base covered a 9 year interval, a total of 4 independent 8 quarter base periods were available in performing our forecast error studies. These base periods are the intervals CY71-72, CY73-74, CY75-76, and CY77-78, respectively. Two of these base periods are illustrated Figure I-5. In performing our error analyses, we began with the CY71-72 base period. For a given item, we computed the average and MAD of demands associated with this base period. Next, we used the appropriate flying program ratio to adjust this average of CY71-72 demands to obtain a forecast of quarterly demands for each quarter in the CY73-CY74 interval. That is, the data observed in quarters 1-8 were used to forecast demands for each quarter in the interval from quarter 9 through quarter 16. We then used the formulas illustrated in Figure I-4 to compute the standardized errors associated with these forecasts. The subscript i associated with the standardized error Zi refers to a forecast of i periods into the future. Consequently, when the time interval CY71-72 is the base period, Z1 refers to the forecast error associated with a quarter 9 forecast; i. e. a forecast 1 time period into the future. Similarly, Z2 refers to the standardized period error associated with a forecast for quarter 10, a forecast of 2 quarters into the future. Similar definitions apply to the symbols LTZi.

Once the standardized errors associated with each lead time i were computed for the first base period, the standardized error calculations were repeated using the period CY73-74 as the base period. In this second case, the data from quarters 9-16 were used to forecast the quarterly demands for quarters 17-24. Again the standardized errors Zi were computed and tabulated. In this case, Z1 refers to the forecast for quarter 17, while Z2 refers to the standardized error for the quarter 18 forecast. Once the CY73-74 base period calculations were completed, similar calculations were performed

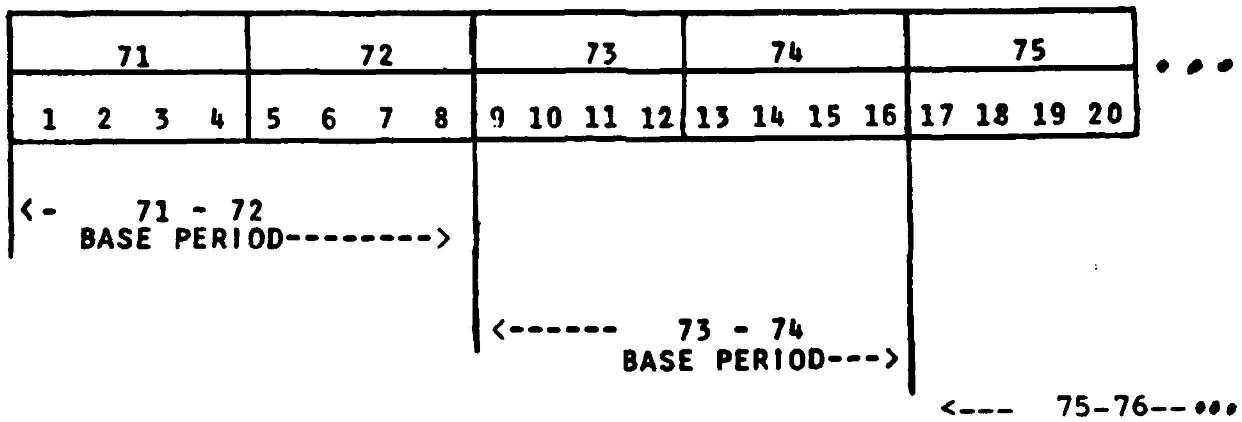


Figure I-5. Numbering Conventions for Data Period.

and tabulated for the base periods CY75-76 and CY77-78. Detailed plots and historgrams for the standardized errors were then produced from these calculations and other forms of statistical analyses of the standardized errors were performed. Similar activities were performed in analyzing the lead time demand forecast errors LTZi.

The Current D062 Model for Forecast Errors

At present, D062 safety level calculations are based upon the assumption that the distribution of forecast errors for demands in a leadtime is normally distributed. This assumption is based on the fact that if a large number of identically distributed random variables are summed, the sum will tend to a normal distribution regardless of the underlying distribution of the individual random variables. This is a very common assumption in inventory management systems, and simulation studies using actual D062 demands have shown that the safety level formulas based upon these assumptions provide significantly more cost effective policies than formulas previously used in the D062 system. Rather than use the normal probability distribution directly, however, D062 safety level calculations utilize the Laplace distribution to approximate the normal. Presutti and Trepp (1970) observed that the unit normal and unit Laplace distributions are very close approximations to one another, and they illustrate this fact by comparing the cumulative probability distributions shown in Table I-1. Figure I-6 illustrates the probability density function for the Laplace distribution. Notice that for this distribution most observations are concentrated about the mean, but the distribution has a fairly long tail. Figure I-7 compares the Laplace distribution with the normal probability distribution. Note that when compared to the Normal distribution, the Laplace has more probability in the tails of the distribution as well as a slightly higher probability that observed demands will be very close to the mean.

Table I-1. A Comparison of the Unit Laplace and Unit Normal Distribution Functions.*

k	A^a	B^b	$C_n(x)^c$	$C_f(x)^c$
0	0.0793	0.1232	0.0793	0.1232
0.2	0.0761	0.0928	0.1554	0.2160
0.4	0.0703	0.0700	0.2257	0.2860
0.6	0.0624	0.0527	0.2881	0.3387
0.8	0.0532	0.0397	0.3413	0.3784
1.0	0.0436	0.0299	0.3849	0.4084
1.2	0.0343	0.0226	0.4192	0.4310
1.4	0.0260	0.0170	0.4452	0.4480
1.6	0.0189	0.0128	0.4641	0.4610
1.8	0.0132	0.0097	0.4773	0.4704
2.0	0.0088	0.0073	0.4861	0.4777
2.2	0.0057	0.0055	0.4918	0.4832
2.4	0.0035	0.0041	0.4953	0.4874
2.6	0.0021	0.0031	0.4974	0.4905
2.8	0.0013	0.0024	0.4987	0.4928
3.0	0.0006	0.0018	0.4993	0.4946
3.2	0.0004	0.0013	0.4997	0.4959
3.4	0.0001	0.0010	0.4998	0.4969
3.6	0.0001	0.0008	0.4999	0.4977
3.8	0.0001	0.0006	0.5000	0.4982
4.0	0.0000	0.0004	0.5000	0.4987

[a] $A = Pr\left[k \leq \frac{x-\mu}{\sigma} \leq k+0.2\right]$ for $n(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$.

[b] $B = Pr\left[k \leq \frac{x-\mu}{\sigma} \leq k+0.2\right]$ for $f(x) = \frac{\sqrt{2}}{2\sigma} \exp\left(-\sqrt{2}\left|\frac{x-\mu}{\sigma}\right|\right)$.

[c] $C(x) = Pr\left[0 \leq \frac{x-\mu}{\sigma} \leq k+0.2\right]$ for $n(x)$ and $f(x)$, respectively.

*From Presutti, Victor J. and R.C. Trepp, "More Ado about EOQ," Naval Research Logistics Quarterly, v17,n2,June 1970, p.245.

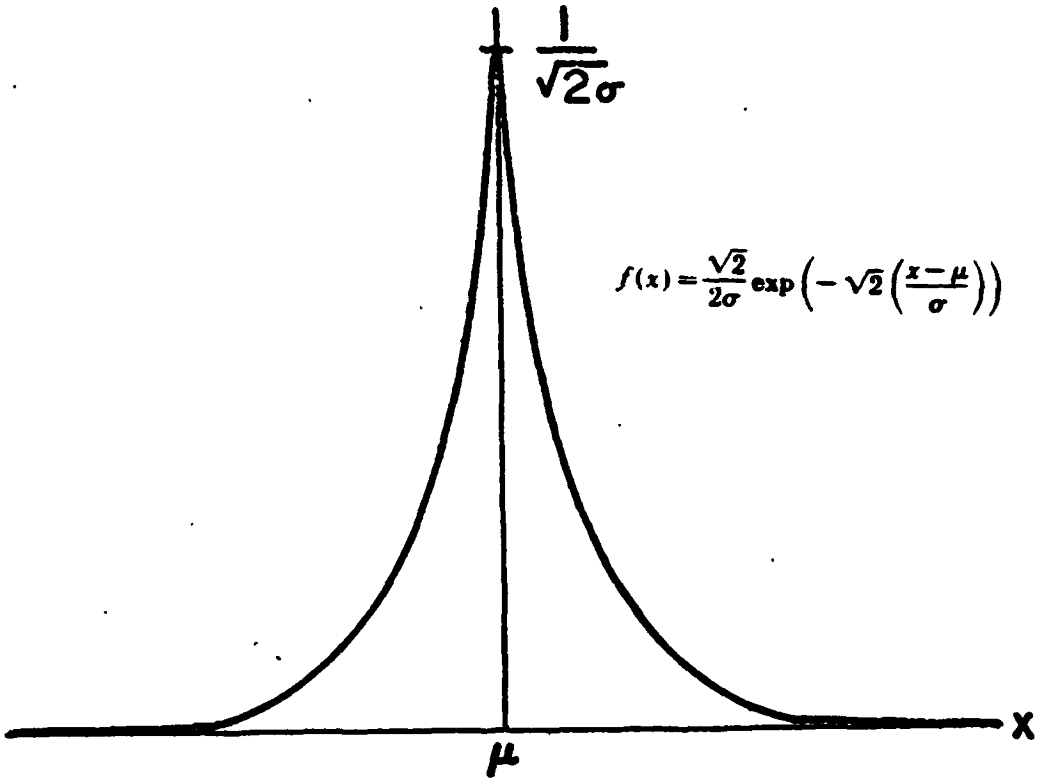


Figure I-6. The Laplace probability density function.

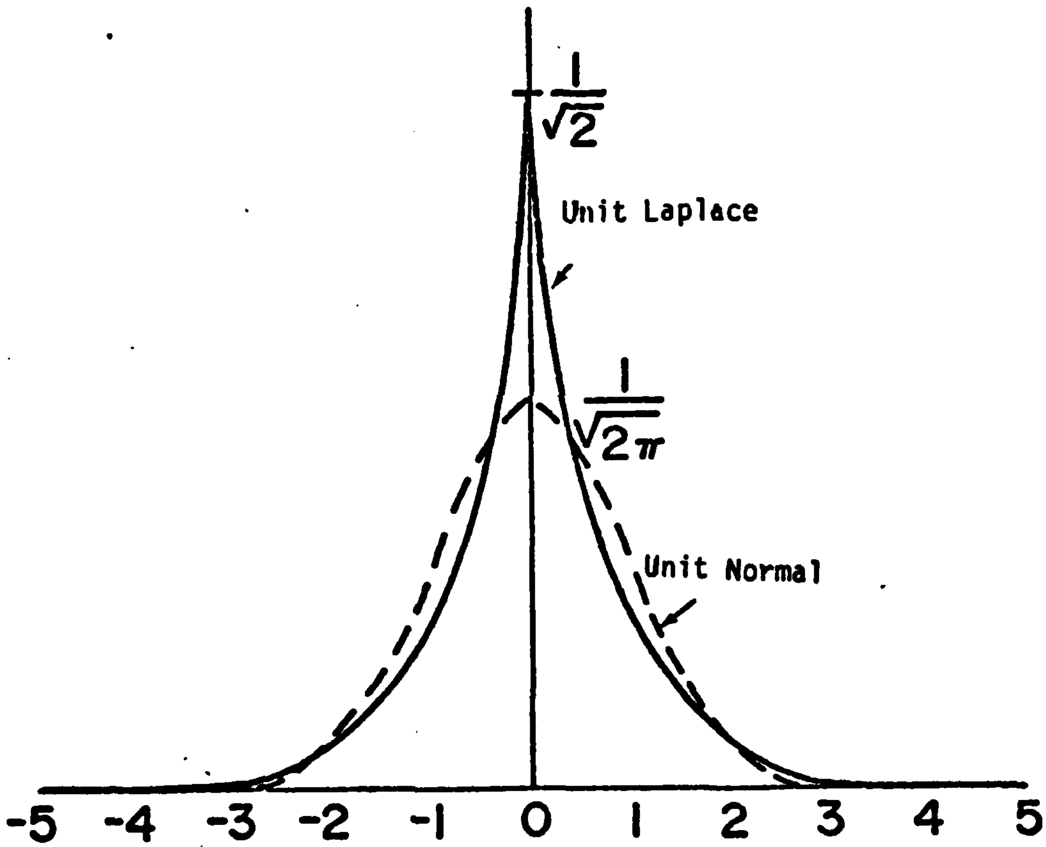


Figure I-7. A comparison of the unit normal and unit Laplace ($\mu=0,\sigma=1$).

Figure I-8 compares the cumulative distribution function for the normal probability density with the observed Zi statistics from a Sacramento ALC item sample. As shown in the figure, the cumulative normal distribution will pass through the fiftieth percentile when the standardized error Z = 0. Also observe that is extremely unlikely for a standardized error to exceed 4 MADS above the mean of the normal distribution. On the other hand, Figure I-8 also illustrates the distribution of period errors Zi observed by Demmy (1979). This figure presents the observed distributions of Zi for period errors in making forecast which are 1, 2, 3, and 4 quarters in the future. These particular curves were derived from an analysis of a low activity SA-ALC demand sample utilizing CY1971 to 72 is the base year for the forecast calculations. However, almost identical curves were observed using other Air Logistics Center samples and other base periods.

If demand per quarter were in fact normally distributed, the observed distribution of forecast errors should exactly correspond with the theoretical normal curve illustrated in Figure I-8. Obviously, there is a significant discrepancy. One explanation for this discrepancy is that the demands per period is in fact described by a stationary, but skewed probability distribution. Another explanation is that the underlying demand process is in fact stationary but subject to substantial trends, with some items having positive or increasing trends and other items having negative (decreasing) trends. In this case, a "straight-line" forecasting technique would result in a distribution of forecast errors similar to that presented in Figure I-8. This explanation was suggested by Hayya (1980). Still another situation that would produce Zi curves similar to those observed is that demand is a skewed but non-stationary distribution.

C.D.F. OF ZI FOR 1971-72 BASE YEAR

FOR Z1-Z4 AND NORMAL FOR SMALC SMGC=1

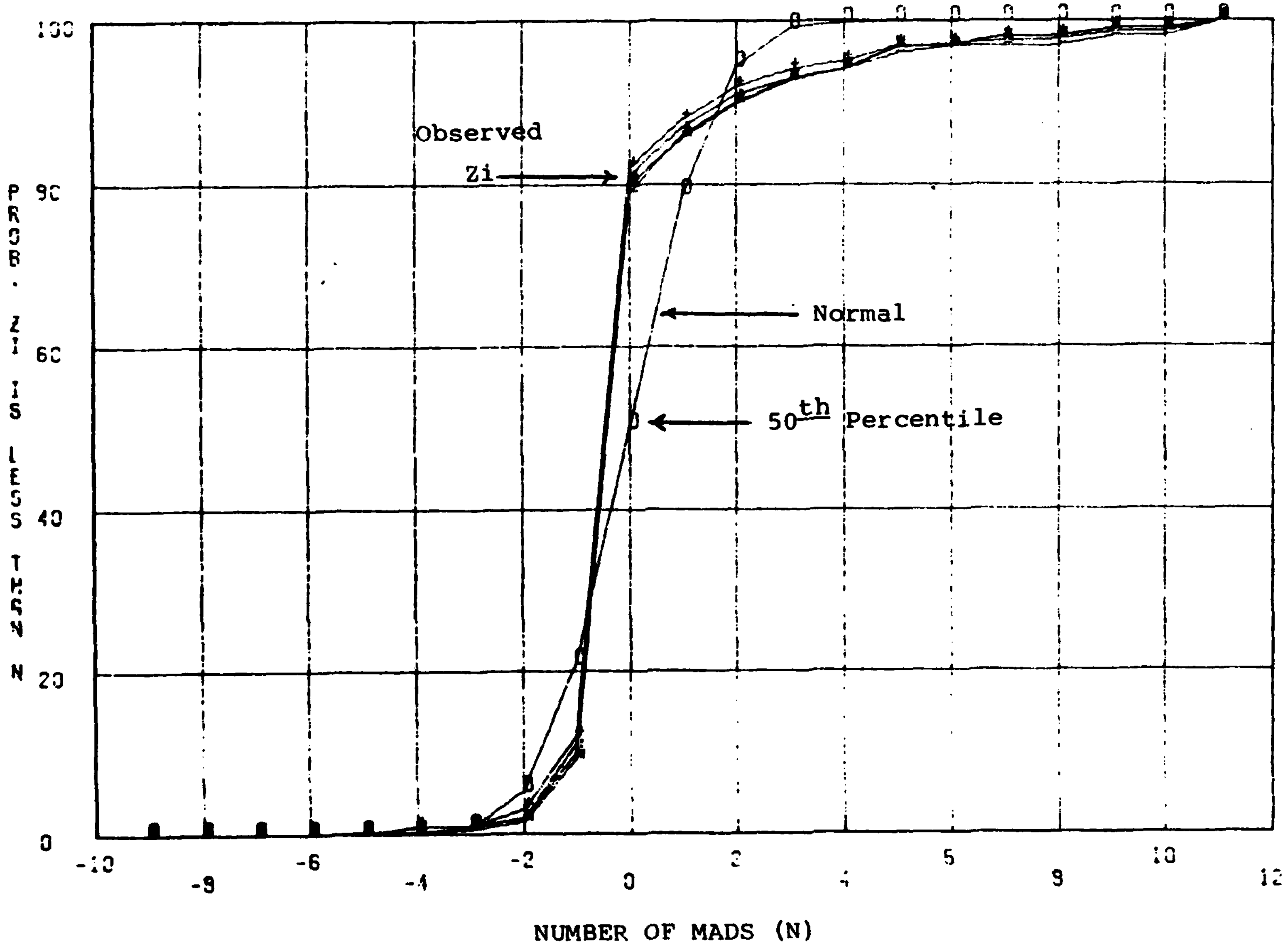


Figure I-8. An Observed Distribution of Period Errors.

The major objective of the studies reported in this paper was to obtain a better understanding of the true nature of the underlying standardized error distributions for Zi and LTZi. In the next section, we report the results of the preliminary data analysis required to obtain a better understanding of the empirical data.

Section II

Initial Results

At present, D062 safety level calculations assume that the same probability distribution--the Laplace--is a useful approximation to the distribution of forecast errors LTZi for all EOQ items. To test the accuracy of this assumption, we performed a number of statistical studies using the D062 demand histories from the INSSIM Data Bank. We were particularly interested in determining how the distribution of forecast errors LTZi differed among time periods and item groupings. For example, were the forecast error distributions observed in the early 1970s similiar to those observed during the later part of the decade? Are there significant differences in forecast accuracy across aircraft types? Or are there significant differences in forecast accuracy across other types of item groupings. If no differences exist, a single probability distribution is appropriate for the performance of safety level calculations, and the large amount of historical data present in the INSSIM data bank may be used to develop a precise description of this distribution. On the other hand, if significant differences exist, improved safety level calculations might be developed by using the specific probability distribution which is appropriate for the different item classes.

Plots of Item Demand Histories

We began our study by developing plots of the actual demand histories of several hundred items from the INSSIM data bank. We then visually compared these patterns both across items and with the flying programs of the primary aircraft associated with each item. In doing this, we were primarily interested in obtaining answers to the following types of questions:

1. What do D062 item demand histories look like?

2. Are there noticeable trends in these histories? Are there noticeable cycles or other patterns present?

3. How does this data relate to flying hour programs.

Decisions Systems Working Paper 81-01 (1981) presents plots for 100 Sacramento-ALC items that were studied in this phase, while Figures II-1 thru II-11 illustrate our results. Figure II-1 presents the actual and predicted flying programs for F/FB 111 aircraft for the CY71-79 interval, while Figures II-2 thru II-11 present representative demand histories of items which support the F/FB 111. Figures II-2 thru II-5 are "high" demand items; that is, these items had annual dollar demand rates which exceeded $5,000 per year in the CY 71-72 interval, while Figures II-6 thru II-11 present plots for items which had demands of less than $5,000 per year during this interval.

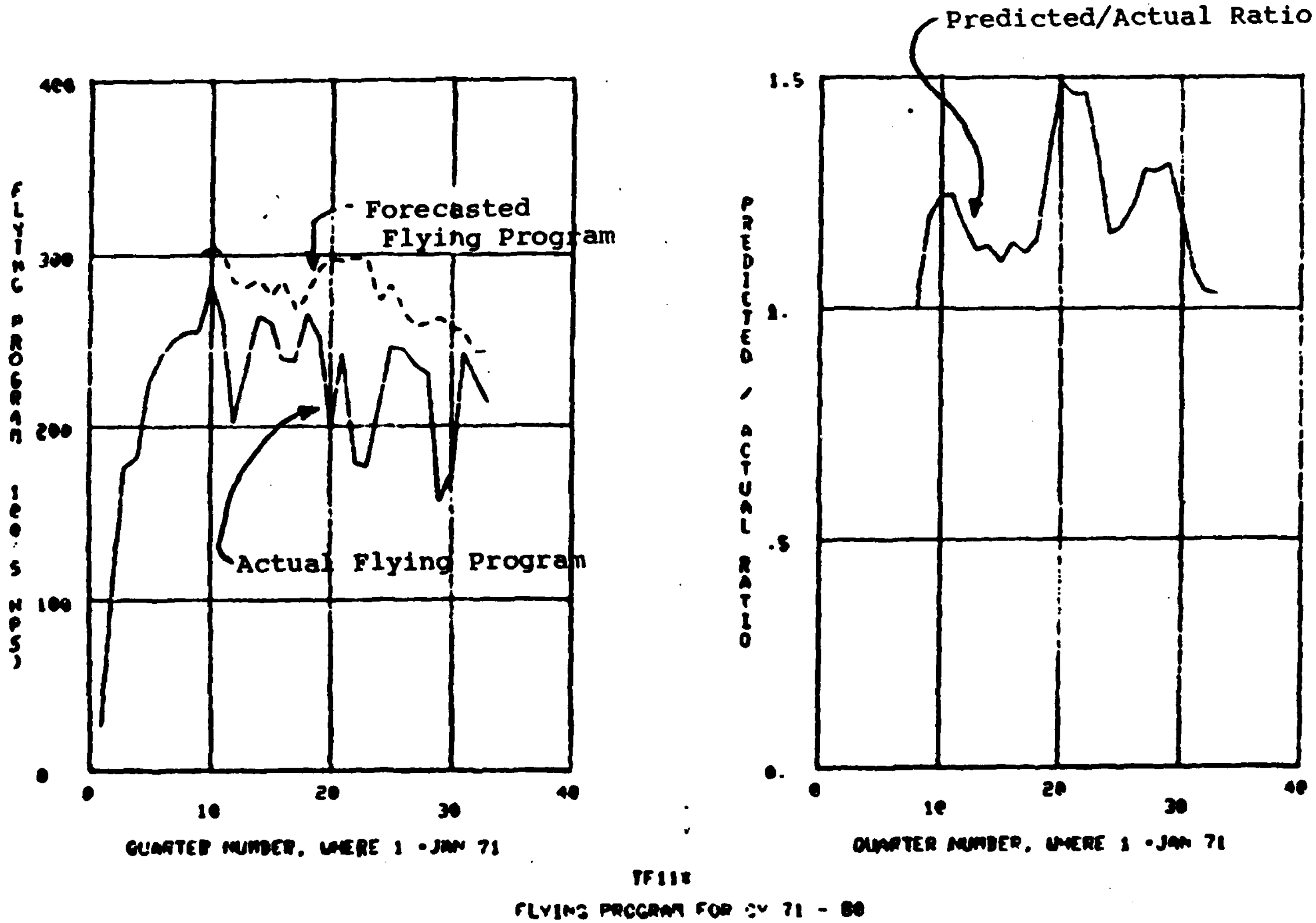


Figure II-1. Actual and Predicted Flying Porgrams for F111 Aircraft for CY71-80.

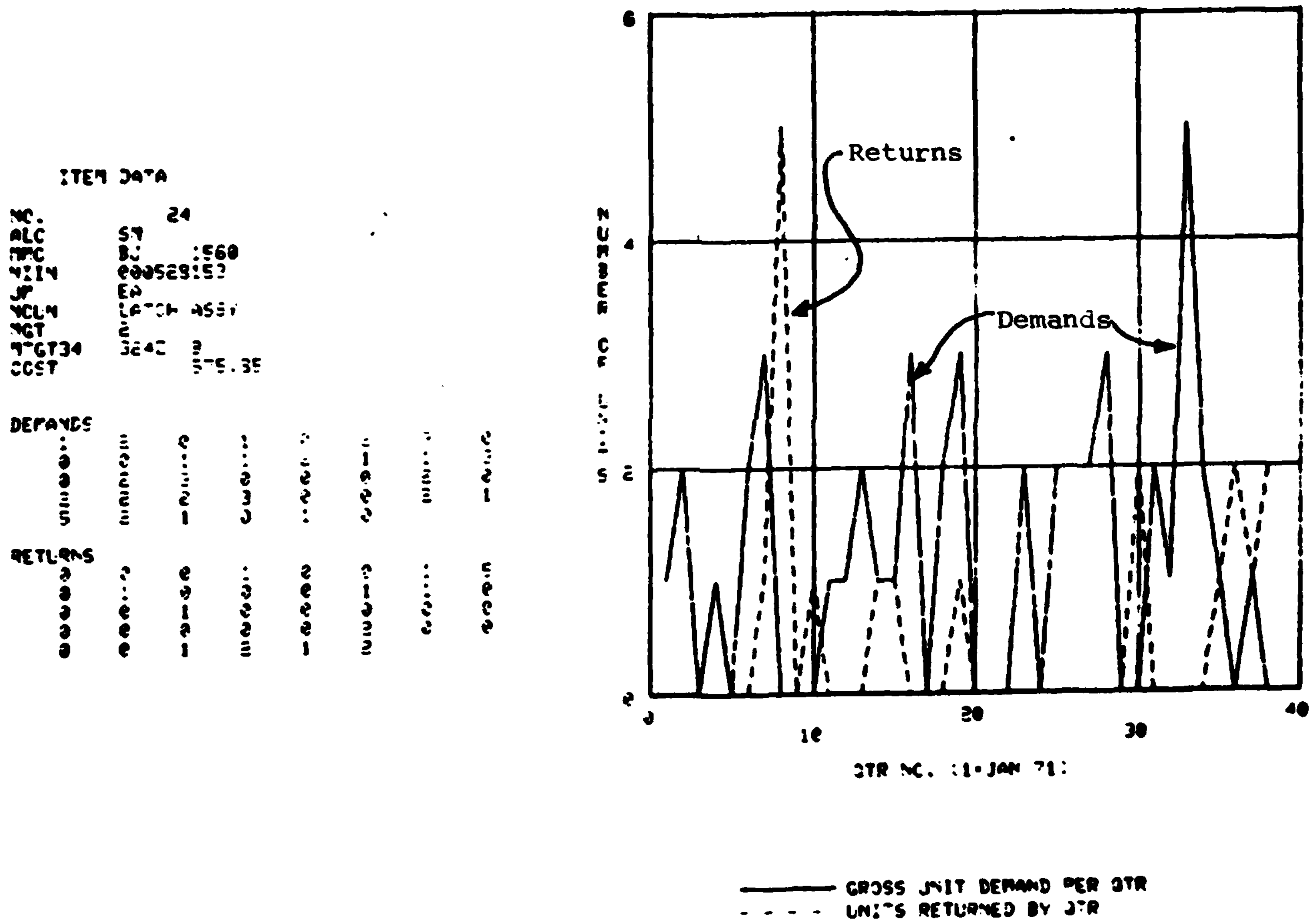


Figure II-2. Observed Demands and Returns for FSN1560 000528153.

ITEM DATA

NO.	1
A.C	SP
PMC	BJ 1055
NIIN	002265032
UM	EA
NOUN	INTRULETTR
PGT	4
PTS 34	3242 B
COST	805.85

DEMANDS

4	[illegible]	4	[illegible]	3	[illegible]	11	[illegible]
11	23	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6	5	[illegible]	[illegible]	[illegible]	[illegible]	4	[illegible]
8	12	[illegible]	[illegible]	15	[illegible]	1	[illegible]
[illegible]	11	[illegible]	[illegible]	1	[illegible]		

RETURNS

0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0	[illegible]	4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		

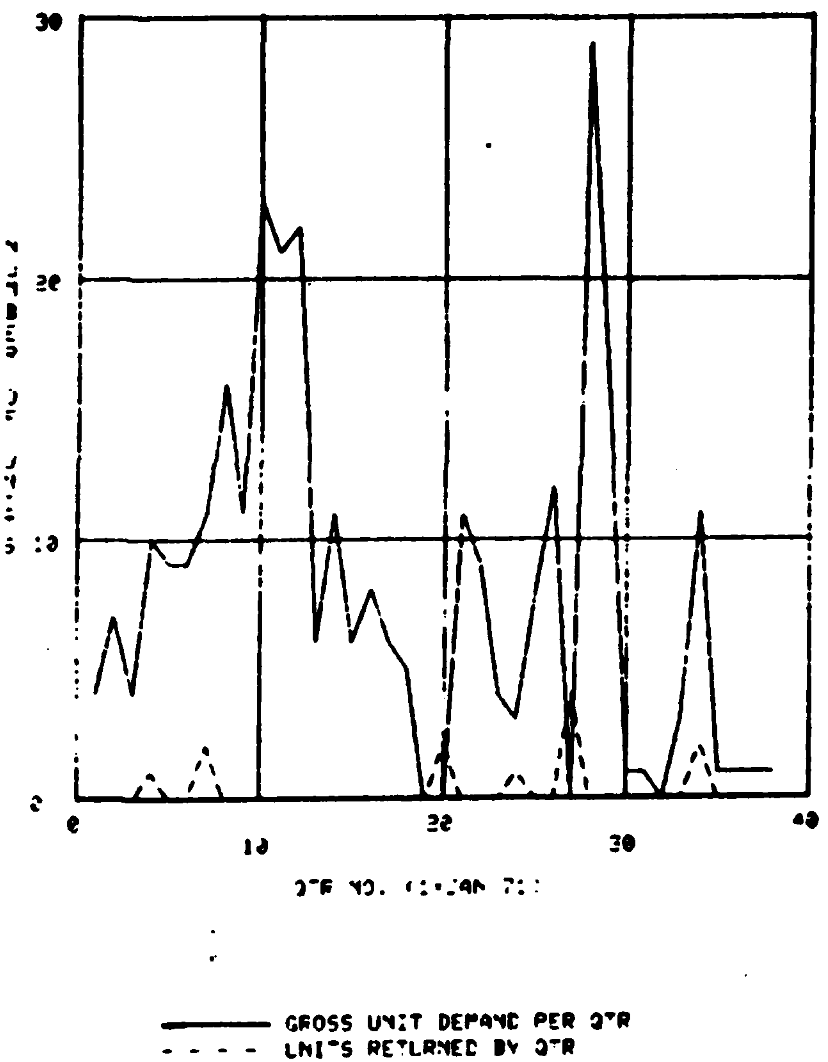


Figure II-3. Historical Demands and Returns for a High Activity F/FB111 Item.

ITEM DATA

NO.	25
ALC	SM
MMC	BJ 1560
NIIN	00074351:
UM	EA
NOUN	BRK ASSY
PGT	4
PTS734	3242 B
COST	203.95

DEMANDS

0	6	10	10	3	6	1	0
3	3	0	6	1	0	2	[illegible]
0	0	2	[illegible]	0	2	0	2
0	0	0	0	[illegible]	2	0	5
0	1	0	0	2	2		

RETURNS

0	0	[illegible]	0	0	2	2	0
2	0	2	4	0	0	3	0
0	0	0	0	0	0	0	2
0	0	[illegible]	0	0	0	0	2
0	0	0	0	0	0		

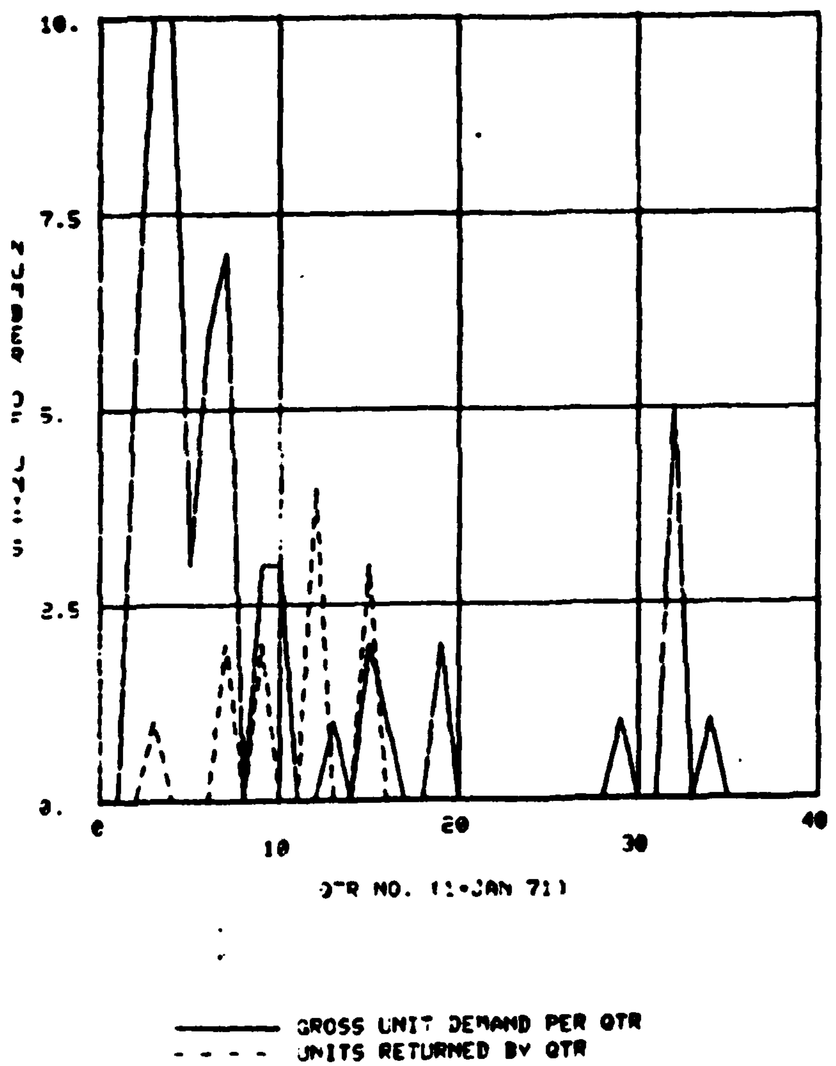


Figure II-4. Historical Demands and Returns for a High Activity F/FB111 Item.

ITEM DATA

```
NO.          26
ALC      SM
MMC      BJ    1560
NIIN     000726461
UM       EA
MCLM     STRAKE
MGT      .
MTGT34   3242  2
COST         2348.32
```

DEMANDS

[illegible]

RETURNS

[illegible]

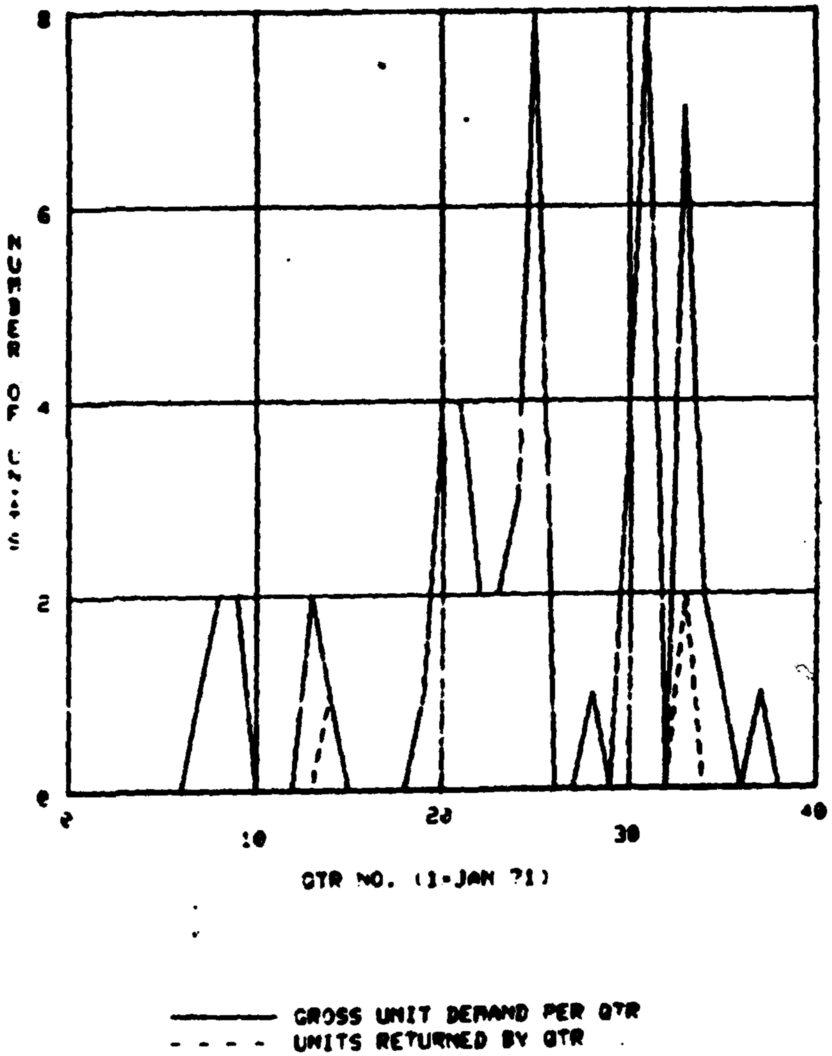


Figure II-5. Historical Demands and Returns for a High Activity F/FB111 Item.

```
    ITEM DATA

NO.             24
ALC         SM
MMC         BJ     1280
NIIN        001193005
UM          EA
NOUN        CIRCUIT CD
MGT         2
MTGT34      3242  B
COST               31.20

DEMANDS
     0     0     4     0     0     0     0     0
     0     0    10     1     0     0     0     0
     0     0     3     7     0     0     0     0
     0     0     0     0     0    20     0     6
     0     1     0     0     0    10

RETURNS
     0     0     0     0     0     3     1     0
     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0
     0     0     0     0     0     0
```

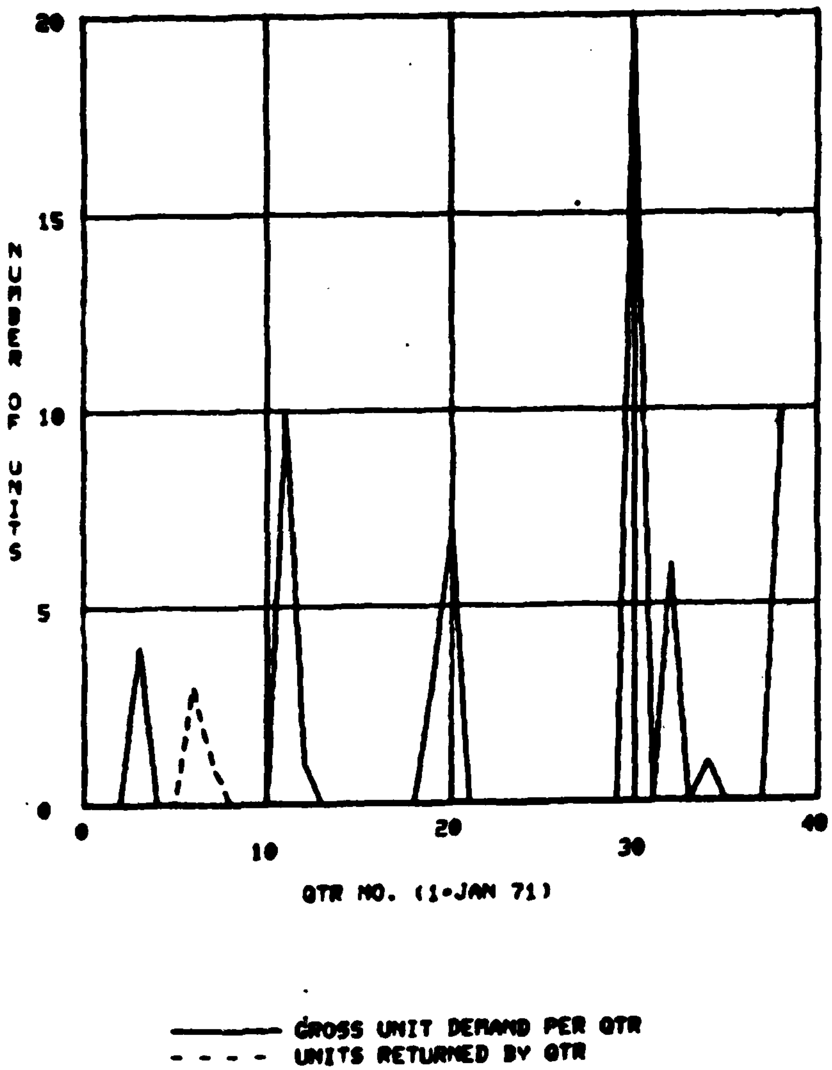


Figure II-6. Historical Demands and Returns for a Low Activity F/FB111 Item.

ITEM DATA

NO. 26
ALC SM
MMC BJ 1280
NIIN 004983350
UM EA
NOUN GUIDE
MGT 3
MTGT34 3242 B
COST 0.94

DEMANDS

0	3	0	60	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0		

RETURNS

0	0	0	0	0	0	0	60
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0		

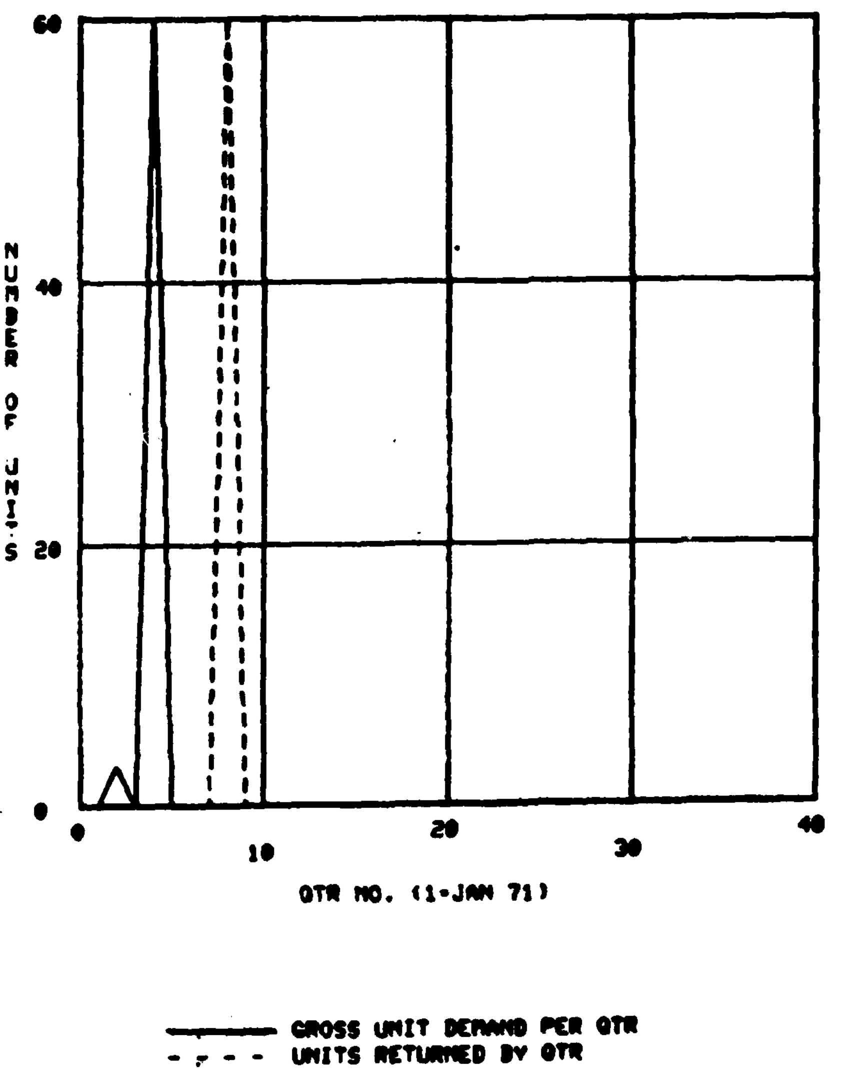


Figure II-7. Historical Demands and Returns for a Low Activity F/FB111 Item.

ITEM DATA

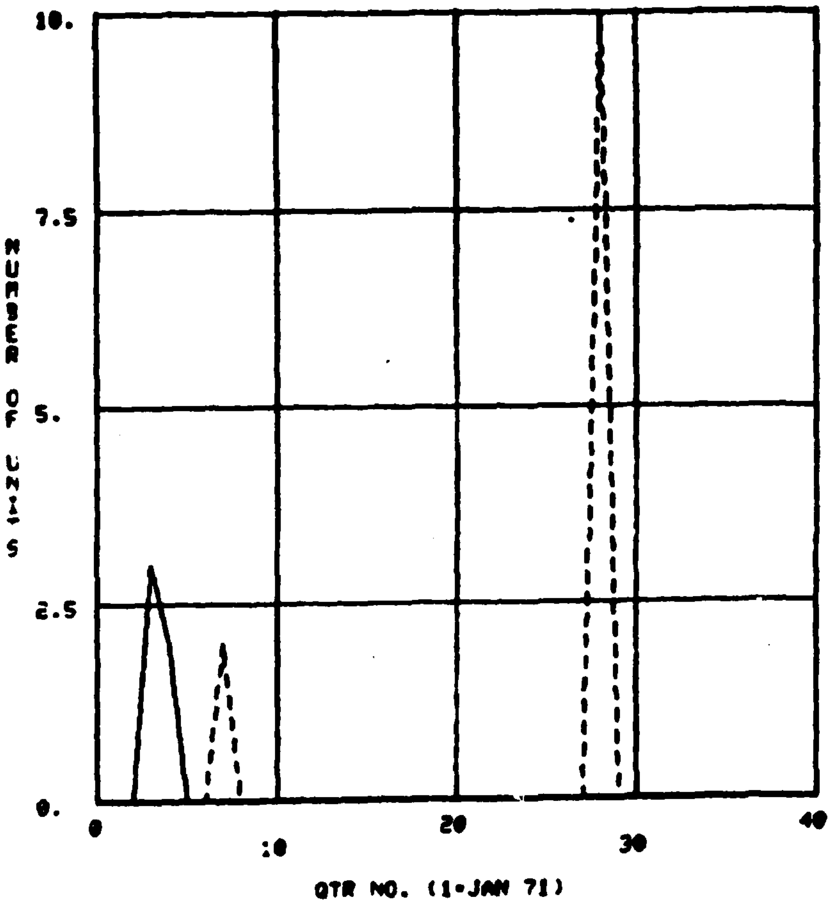
NO.	27	
ALC	SM	
MMC	BJ	1280
NIIN	004988115	
UM	EA	
NOUN	PLATE ASSY	
MGT	3	
FTGT34	3242	C
COST		194.88

DEMANDS

0	0	3	2	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0		

RETURNS

0	0	0	0	0	0	2	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0		



——— GROSS UNIT DEMAND PER QTR
- - - - UNITS RETURNED BY QTR

Figure II-8. Historical Demands and Returns for a Low Activity F/FB111 Item.

ITEM DATA

NO. 32
ALC SM
MMC BJ 1430
NIIN 004563900
UM EA
NOUN HOUSING AY
MGT 2
MTCT34 3242 8
COST 283.92

DEMANDS

0	0	7	3	0	10	0	0
3	3	3	3	0	5	3	2
0	0	20	10	3	2	0	4
3	0	4	5	10	5	10	0
10	29	20	10	0	10		

RETURNS

0	0	0	0	0	10	0	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	9	0	0	0	0
9	0	0	[illegible]	0	0		

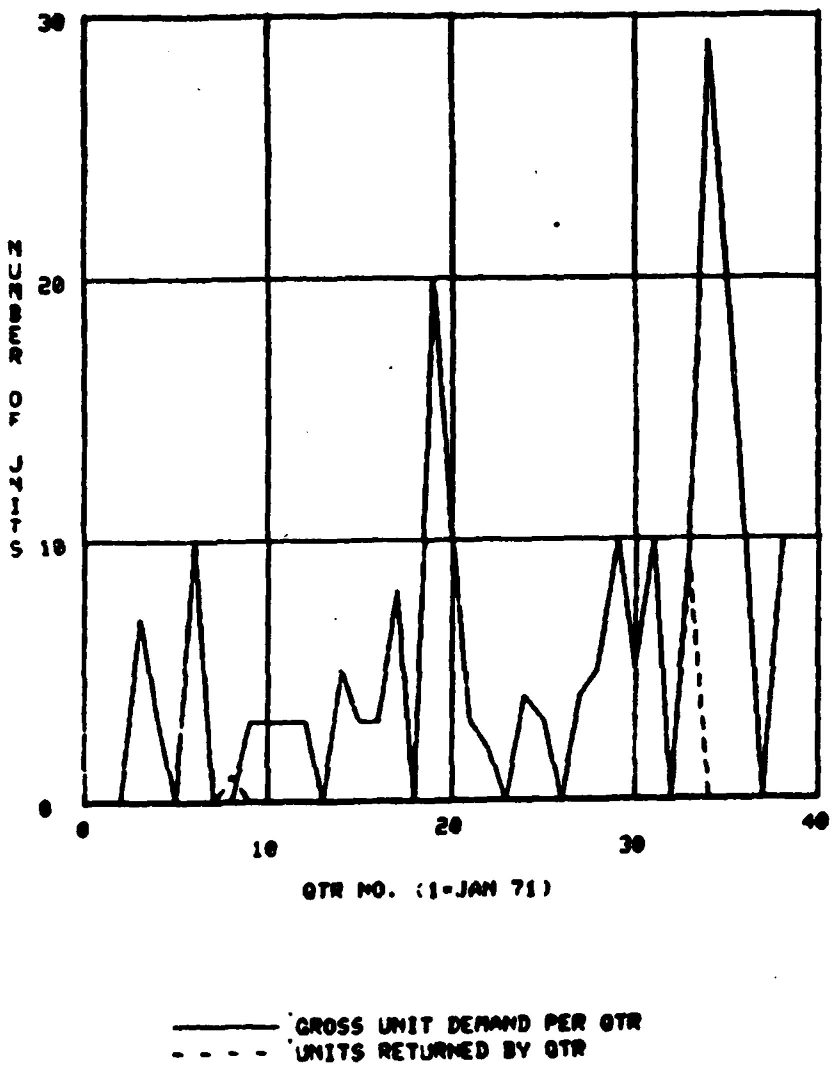


Figure II-9. Historical Demands and Returns for a Low Activity F/FB111 Item.

```
ITEM DATA

NO.             43
ALC        SM
MMC        BQ     1560
NIIN       000180733
UM         EA
NOUN       HINGE
MGT        2
MTGT34     3242  B
COST              502.53

DEMANDS
    0    0    0    0    0    0    0    0
    0    0    0    0    0    0    0    0
    2    0    0    0    0    0    0    0
    0    0    0    0    0    0    0    0
    0    0    0    0    0    0

RETURNS
    0    0    0    0    0    0    0    0
    0    0    0    0    0    0    0    0
    0    0    0    0    0    0    0    0
    0    0    0    0    0    0    0    0
    0    0    0    0    0    0
```

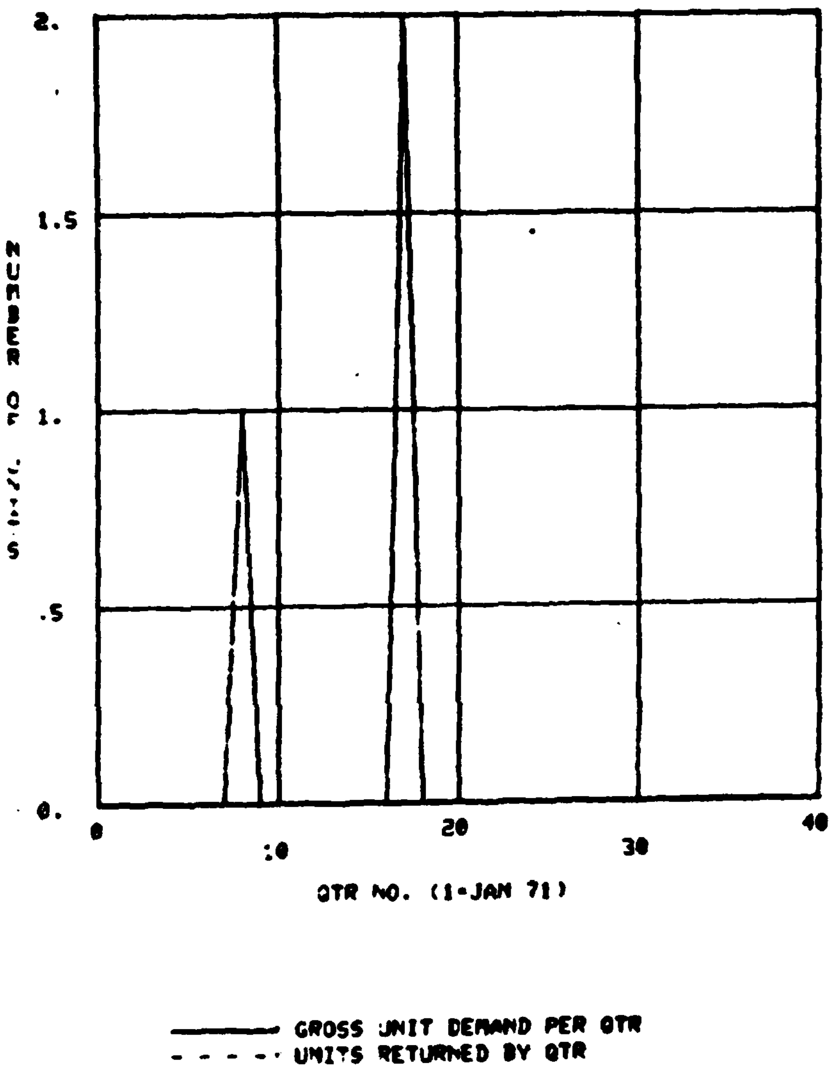


Figure II-10. Historical Demands and Returns for a Low Activity F/FB111 Item.

ITEM DATA

NO.	46
ALC	SM
MPC	BJ :560
NIIN	000230276
UM	EA
NOUN	INSULATOR
MGT	3
MTGT34	3242 B
COST	6.63

DEMANDS

0	17	3	5	6	7	135	0
0	0	12	0	0	0	0	6
0	6	0	2	23	22	40	57
301	143	75	127	165	52	158	52
66	0	46	17	23	153		

RETURNS

0	0	0	0	0	0	0	0
20	12	6	15	0	0	0	48
0	0	0	0	0	0	0	0
20	0	0	0	6	0	0	0
0	0	0	0	0	0		

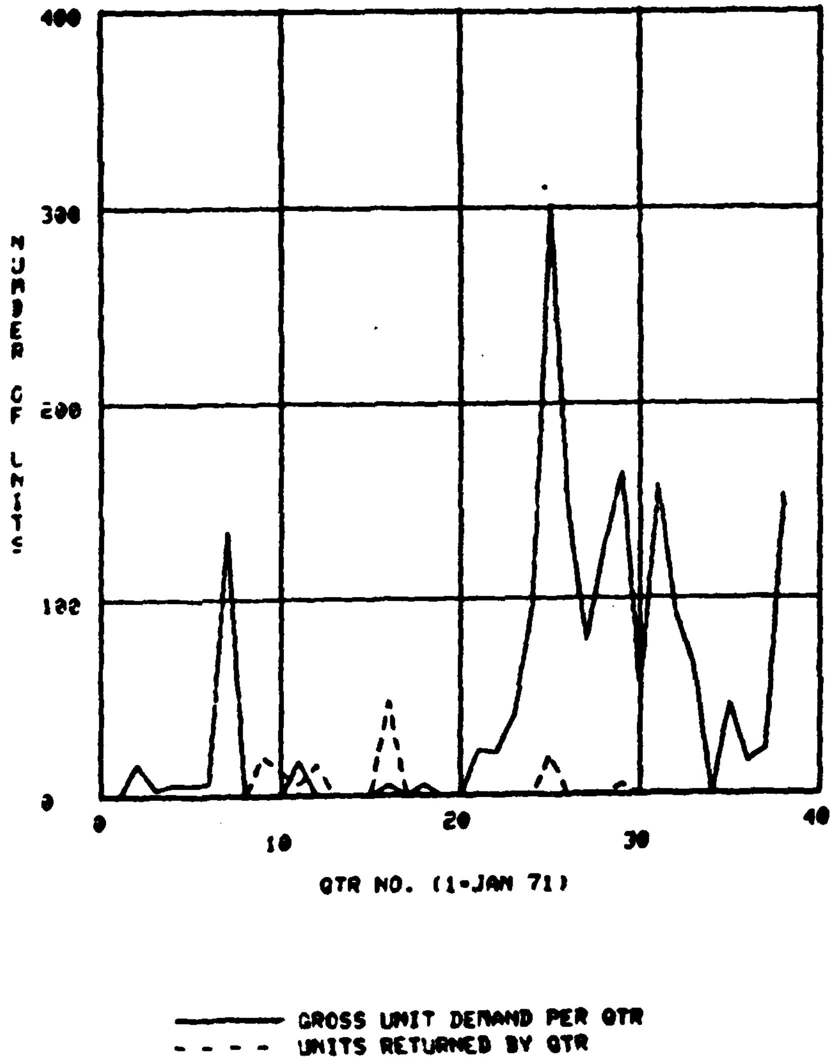


Figure II-11. Historical Demands and Returns for a Low Activity F/FB111 Item.

As shown in II-1, the F/FB111 was being introduced into the Air Force inventory during the beginning of the 1970s. The flying program for this aircraft increased during the first two and a half years of the decade, and has been undergoing a slight downtrend since that time. As shown on the right hand side of Figure II-1, Air Force G033J forecasts of flying activity for the F/FB111 have tended to exceed the actual hours flown.

Figure II-2 illustrates the observed demands and returns for FSN 1560 000528153, an item which supports the F/FB 111 aircraft. In this figure, the solid line denotes demands observed each quarter during the CY71-79 interval, while the dotted line denotes serviceable returns recorded by the D062 system. For this item, demands appear fairly stable throughout the nine year time period. However, no noticeable relationship with the actual flying program for the F/FB 111 is present.

Figure II-3 illustrates another type of pattern observed in D062 demand histories. If one takes a large time period view, one might conclude that this pattern represents a stationary but highly erratic demand pattern. On the other hand, if one assumes that the underlying demand process is fairly stable but has a trend that shifts through time, one might conclude that this patten is associated with an item which initially has a significant increasing trend during the first three years of the time interval, followed by a significant decreasing trend during the next three year interval. The final demand values

might be interpreted as fairly stationary demands with a spike in quarter 28, -- i.e., a large unexpected demand inconsistent with the underlying pattern. For this item, observe that if one assumes trends are present in the data, these trends bear no clear relationship with the flying program activities for the F/FB111.

Figure II-4 illustrates another pattern found in D062 demand histories. This item experiences a significant downtrend during the CY71-75 interval. It is followed by nine quarters of zero demand and then another time interval in which there are demands in three of the six subsequent quarters. Again, the demand histories for this item bear no clear relationship to the flying program for the F/FB111.

Figures II-5 presents still another pattern. In this case demands are quite erratic with a long term increasing trend; however, a sharp drop in the level of demand occurs in the last few quarters of the 38 quarter interval.

Figures II-6 thru II-11 present representative demand histories for "low" activity items, i,e., items which had demands of $5,000 per year or less during the CY71-72 interval. The vast majority of D062 items fall into this catagory. Observe that demands for these items are much more erratic--and therefore much more difficult to predict--than the demands for the high

activity items discussed above. In general, demands for these items contain a large number of periods with no observed demands. When demands do occur, they often involve quantities significantly different than one unit. For example, Figure II-7 shows only two quarters out of thirty-eight in which demands are greater than zero. One of these demands was for 60 units. We hypothesize this pattern was due to a requisition which was improperly coded, for notice that 60 units were returned one year after the 60 units of demand were placed on the D062 system.

After reviewing a very large number of plots for D062 items, it was difficult to distinguish any single pattern which appeared useful in forecasting. For some items, demand does in fact appear to follow flying program activity. For other items, apparent trends in demand appear to be unrelated to program activity. For still other items, no consistent trends were observed. The only pattern which appeared common across large numbers of D062 items was the presence of spikes--i.e., the presence of very large and unexpected demands which are inconsistent with demand activity both before and after the occurance of the spike. Spikes are particularly bothersome for low activity items. Unfortunately, spikes appear in the patterns of a large number of D062 demand histories.

The Distribution of Period Errors Zi

In this section, we discuss our preliminary analysis of the distribution of period errors Zi. Of particularly interest in this section are the following questions:

Are there significant differences in the distribution of period errors Zi

(a) across base periods?

(b) across lead times?

(c) across Air Logistics Centers?

(d) across dollar demand classess?

To answer these questions, we built four item samples which we denote as OC.H, OC.L, SM.H, and SM.L, respectively. Sample OC.H represents a sample of INSSIM Data Bank high activity items managed by Oklahoma City Air Logistics Center, while sample OC.L represents a sample of low activity Oklahoma City D062 items. Similarily, samples SM.H and SM.L were obtained from the INSSIM Data Bank histories for Sacremento Air Logistics Center. To be included in a "high" activity category, an item must have

had an average of at least $5,000 of demands per year during the CY71-72 inteval; otherwise, the item was classed in the low demand category.

As noted in section I, a total of 38 quarters of data were available for the study. This permitted us to compute the forecast errors associated with four independent two-year base periods. The base periods involved were the intervals CY71-72, CY73-74, CY75-76, and CY77-78. As discussed in chapter I, we used the demands observed in a given base period to compute the mean and MAD associated with these demands. This information was modified for forecasted changes in flying activity to obtain the forecast demand in each future quarter. The standardized error Zi represents the ratio (observed demand - forecasted demand)/MAD, where the observed demand is associated with the period i quarters beyond the last quarter of the base period. For example, in obtaining forecast associated with the base period CY71-72, the symbol Z2 represents the standarized error associated with demands observed in the second period of CY73.

To obtain an understanding of the shape of the distribution of forecast errors Zi, we computed the standardized errors Zi associated with each item sample and each demand period, and for forecasts of i=1,2,...,12 quarters in the future. Plots of the cumulative distributions associated with each of the error statistics Zi were then obtained and studied in detail. In addition, we used the error statistics Zi to perform Analysis of

Variance (ANOVA) studies to identify differences among different item groupings. These ANOVA studies sought to determine if there are any significant differences in the individual Zi statistics across aircraft, base periods, or demand classes.

The detailed ANOVA tables obtained from these studies are presented in Appendix A.

We found significant differences in the error characteristics associated the CY75-76 base period relative to the other base intervals. In addition, error statistics associated with F104 and F5 aircraft appeared to be of significantly greater magnitude than those associated with the other twenty-one aircraft in the INSSIM data bank. A possible explanation for these significant differences is perhaps explained by the data presented in Figures II-12 and II-13. Figure II-12 presents actual and forecasted flying for the F104 during CY71-79 interval. Note that this weapon system phased out of the Air Force inventory about the middle of the period, but that foreign military sales resulted in increased programs during the latter part of the 1970's. Notice that particularly severe errors are associated with the forecast program accuracy in quarter twenty. This particularly bad forecast of future flying program activity would significantly bias forecasts associated with that period. Now observe the predicted and actual flying programs for the F5 presented Figure II-13. As shown in the figure, the F5 experienced a continuously increasing program throughout the 1970's. The F5 is the only

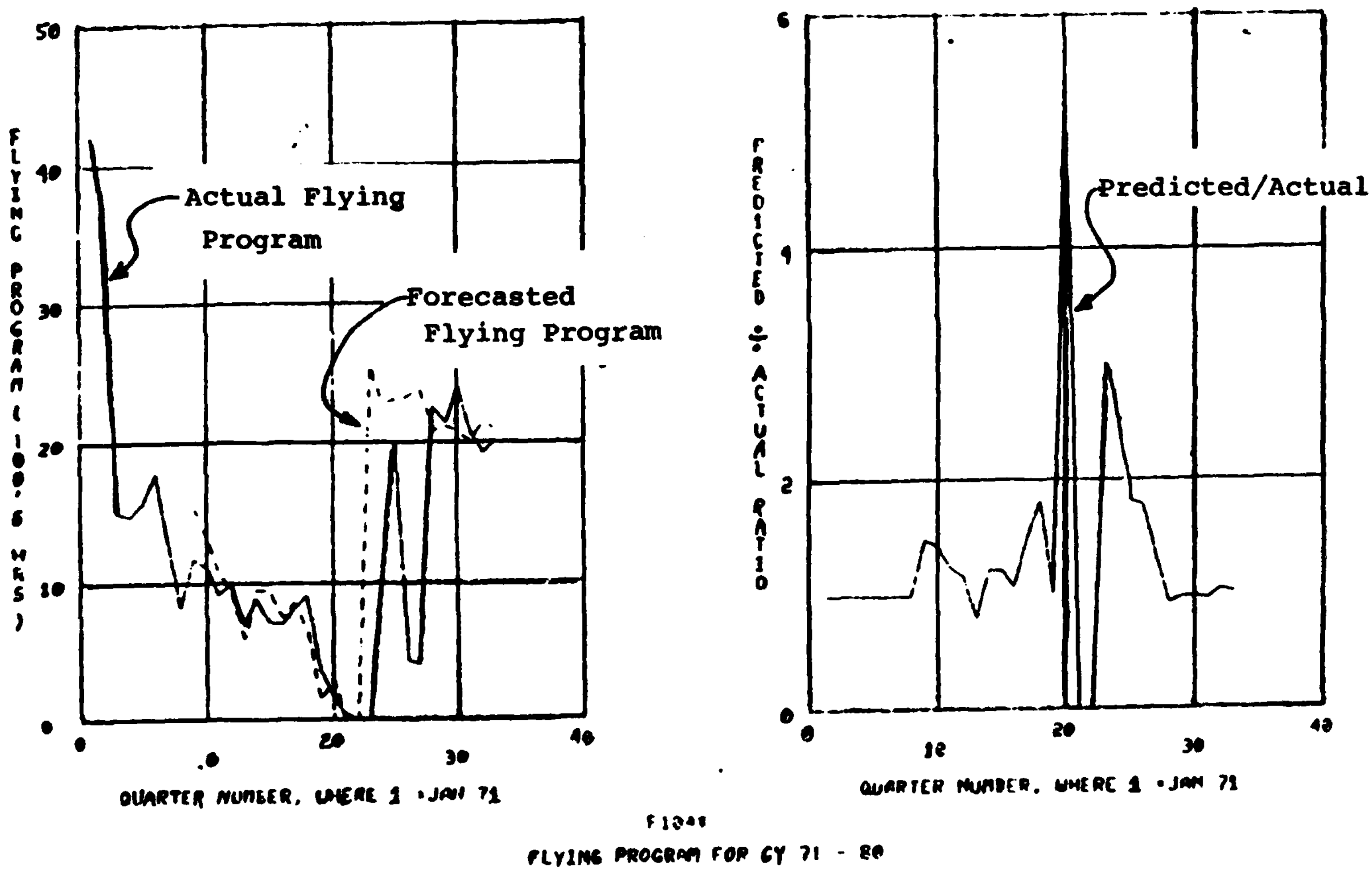


Figure II-12. Predicted and Actual Flying Programs for the F104.

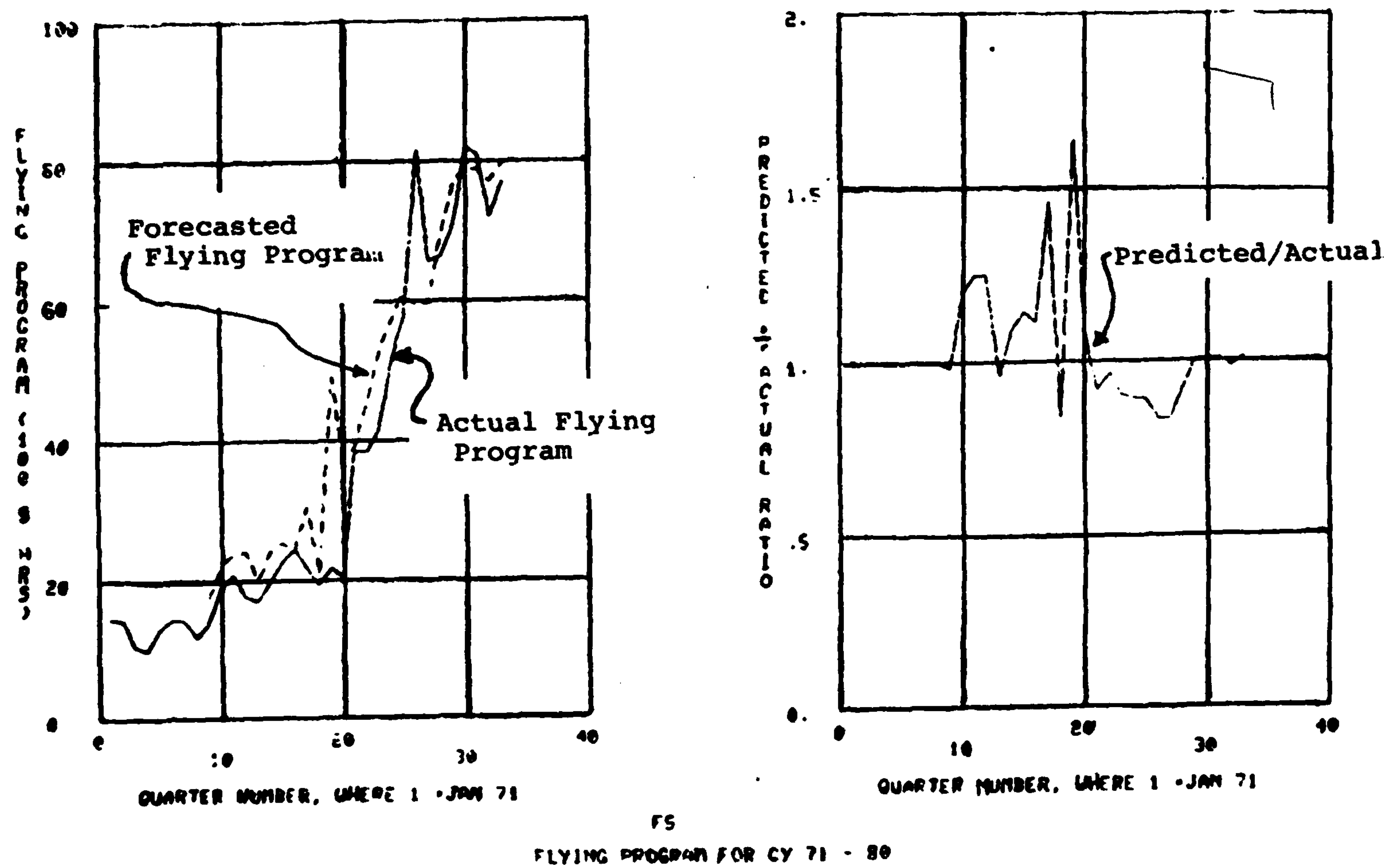


Figure II-13. Predicted and Actual Flying Programs for the F5.

aircraft in the INSSIM data bank which has an ascending program. (Note: the F/FB111 has an increasing program during CY71-72, but declines gently during the remainder of the 70s). Errors in forecasting future flying program activity for the F5 are also very significant during the 75-76 time period.

Based on the above data, it appeared that demand data associated with F-104 and F-5 aircraft would differ significantly from that associated with other aircraft. Consequently, all items associated with these aircraft were deleted from future forecast studies. We then repeated the Analysis of Variances studies for differences across aircraft and across base periods, but this time we excluded F-104 and F-5 items. With these items removed, there were very small differences among the mean forecast errors both by aircraft and base period. Since thousands of forecast were involved, these small differences were still statistically significant, but they were of such small magnitude as to have little practical usefullness. For example, the proportion of variation in mean forecast errors explained by differences across aircraft and base periods are generally much less than one percent for all Zi values studied and never exceeds two-percent. Hence, it appears that differentiating with respect to either aircraft or base periods would provide little improvement in estimating mean forecast errors.

We also performed Analysis of Variance studies to determine if there were any significant differences in forecast errors

based upon the level of activity observed during the base period. The detailed ANOVA tables produced in this effort are presented in Appendix A. We found, yes, there are significant differences in forecast errors based upon the observed base period demands. If little or no demand is observed in the base period, actual demands which occur in future periods tend to be greater than the forecast. On the other hand, if very high levels of demand are observed in the base period, future demands tend to be less than the forecast. In fact, it appeared that the higher the demand forecast (i.e. the higher the demand observed in the based period), the greater the tendency to overforecast future demands. In section III, we will futher investigate differences among forecast error distributions based upon demand classes.

The Analysis of Variance studies indicate that little improvement in the accuracy of forecasting mean forecast errors would be obtained by differentiating across aircraft or base periods. None-the-less, the shapes of the error distributions might still be significantly different across these item groupings. To investigate this issue, we again developed the empirical distributions of period errors Zi associated with each item sample and each base period. This time, however, all items associated with F-104 and F5 aircraft were deleted. Tables II-1 and II-4 and Figures II-14 through II-21 illustrate our results for the high activity sample from Oklahoma City ALC. Similar data for the OC.L, SM.H, and SM.L samples are presented in Appendix B.

Table II-1. Cumulative Probabilities for Period Errors Z for 1971-72 Base Year Forecasts With General Program Factors for Sample OC.H

PERCENTAGES

		1	2	3	4	5	6	7	8	9	10	11	12
1	-9.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
2	-8.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
3	-7.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
4	-6.0	0.	0.	0.	1.	0.	0.	0.	1.	0.	0.	0.	0.
5	-5.0	0.	0.	1.	1.	0.	1.	0.	1.	0.	0.	1.	1.
6	-4.0	1.	1.	1.	1.	1.	1.	2.	2.	1.	1.	1.	1.
7	-3.0	2.	2.	3.	3.	3.	3.	4.	4.	3.	4.	3.	3.
8	-2.0	6.	7.	7.	8.	8.	9.	10.	10.	8.	9.	9.	9.
9	-1.0	24.	26.	27.	28.	30.	33.	34.	34.	30.	34.	32.	33.
10	0.	60.	61.	62.	65.	68.	71.	72.	74.	67.	69.	70.	70.
11	1.0	78.	80.	80.	83.	85.	86.	86.	88.	84.	85.	85.	85.
12	2.0	89.	90.	90.	91.	92.	93.	93.	93.	92.	92.	91.	91.
13	3.0	94.	94.	94.	95.	96.	96.	96.	96.	95.	95.	94.	94.
14	4.0	96.	96.	96.	97.	97.	97.	98.	98.	97.	96.	96.	96.
15	5.0	98.	97.	98.	98.	98.	98.	99.	99.	98.	97.	97.	98.
16	6.0	98.	98.	98.	99.	99.	99.	99.	99.	99.	98.	98.	98.
17	7.0	99.	99.	99.	99.	99.	99.	99.	99.	99.	98.	98.	99.
18	8.0	99.	99.	99.	99.	99.	99.	100.	100.	99.	99.	99.	99.
19	9.0	100.	99.	100.	100.	100.	99.	100.	100.	99.	99.	99.	99.
20	10.0	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.
TOTAL OBS		3153	3153	3153	3153	3153	3153	3153	3153	3153	3153	3153	3153

TABLE	MEAN	VARIANCE	C.OF V.	SKEWNESS	KURTOSIS
1	0.075	4.107	27.135	3.018	27.485
2	0.035	5.024	63.409	4.506	51.473
3	-0.054	4.356	38.393	2.959	33.743
4	-0.164	5.696	14.580	2.095	91.921
5	-0.218	4.886	10.154	6.334	91.134
6	-0.330	4.636	6.518	4.489	53.503
7	-0.384	3.876	5.123	3.288	33.817
8	-0.449	3.992	4.449	4.126	69.519
9	-0.185	4.652	11.686	4.872	64.075
10	-0.203	6.471	12.533	5.063	54.912
11	-0.209	5.626	11.359	3.921	44.933
12	-0.241	5.897	10.090	4.806	67.408

Table II-2. **Cumulative Probabilities for Period Errors Z for 1973-74 Base Year Forecasts With General Program Factors for Sample OC.H**

PERCENTAGES

		1	2	3	4	5	6	7	8	9	10	11	12
1	-9.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0
2	-8.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0
3	-7.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0
4	-6.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0
5	-5.0	0.	0.	0.	0.	1.	1.	0.	1.	1.	1.	1.	1
6	-4.0	0.	1.	1.	1.	1.	1.	1.	1.	1.	2.	1.	1
7	-3.0	1.	2.	2.	2.	2.	3.	2.	2.	2.	3.	3.	2
8	-2.0	5.	6.	6.	6.	6.	7.	7.	6.	7.	7.	7.	7
9	-1.0	21.	24.	25.	24.	23.	24.	25.	24.	26.	24.	27.	26
10	0.	58.	61.	60.	61.	63.	63.	66.	63.	64.	62.	65.	63
11	1.0	78.	81.	80.	80.	80.	80.	82.	79.	81.	78.	82.	80
12	2.0	88.	89.	88.	88.	89.	88.	89.	88.	89.	87.	89.	88
13	3.0	93.	93.	93.	93.	93.	92.	93.	92.	93.	92.	93.	92
14	4.0	96.	96.	95.	95.	95.	94.	95.	94.	95.	94.	95.	94
15	5.0	97.	97.	96.	96.	97.	96.	96.	96.	96.	96.	97.	96
16	6.0	98.	97.	97.	97.	98.	97.	97.	96.	97.	97.	97.	96
17	7.0	98.	98.	98.	98.	98.	97.	97.	97.	98.	97.	98.	97
18	8.0	99.	98.	98.	98.	98.	98.	98.	98.	98.	98.	98.	98
19	9.0	99.	98.	99.	99.	99.	98.	98.	98.	98.	98.	99.	98
20	10.0	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100
TOTAL OBS		3112	3112	3112	3112	3112	3112	3112	3112	3112	3112	3112	311

TABLE	MEAN	VARIANCE	C.OF V.	SKEWNESS	KURTOSIS
13	0.284	8.137	10.034	6.555	92.75[illegible]
14	0.216	9.900	14.562	6.334	70.725
15	0.263	11.022	12.632	7.118	78.349
16	0.250	11.951	13.830	6.118	75.709
17	0.262	14.634	14.619	6.834	69.697
18	0.298	13.566	12.370	5.953	57.840
19	0.198	11.697	17.290	5.937	60.280
20	0.344	13.358	10.615	6.212	59.136
21	0.165	11.749	20.835	6.485	69.490
22	0.293	15.753	13.530	4.884	55.887
23	0.069	12.353	51.216	4.672	71.688
24	0.293	15.018	13.232	5.667	58.508

Table II-3. **Cumulative Probabilities for Period Errors Z for 1975-76 Base Year Forecasts With General Program Factor for Sample OC.H**

PERCENTAGES

		1	2	3	4	5	6	7	8	9	10	11	1[illegible]
1	-9.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	[illegible]
2	-8.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	[illegible]
3	-7.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	[illegible]
4	-6.0	0.	0.	0.	0.	0.	0.	0.	0.	1.	1.	1.	[illegible]
5	-5.0	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	[illegible]
6	-4.0	1.	2.	2.	2.	2.	2.	2.	2.	2.	2.	2.	[illegible]
7	-3.0	3.	3.	3.	3.	4.	3.	4.	4.	4.	4.	5.	[illegible]
8	-2.0	6.	8.	7.	8.	9.	7.	9.	8.	10.	9.	10.	[illegible]
9	-1.0	25.	25.	28.	26.	30.	25.	29.	28.	32.	28.	31.	[illegible]
10	0.	62.	60.	65.	63.	64.	59.	66.	64.	68.	64.	67.	[illegible]
11	1.0	82.	79.	82.	81.	81.	78.	83.	82.	83.	81.	83.	[illegible]
12	2.0	91.	88.	90.	89.	89.	88.	90.	89.	89.	89.	90.	[illegible]
13	3.0	95.	93.	94.	93.	92.	92.	93.	93.	93.	93.	93.	[illegible]
14	4.0	97.	96.	96.	95.	95.	94.	96.	96.	95.	95.	95.	[illegible]
15	5.0	98.	97.	98.	97.	96.	96.	97.	97.	97.	97.	96.	[illegible]
16	6.0	98.	98.	98.	98.	98.	97.	98.	98.	97.	98.	97.	[illegible]
17	7.0	99.	98.	99.	98.	98.	98.	98.	99.	98.	98.	98.	[illegible]
18	8.0	99.	99.	99.	98.	98.	98.	99.	99.	98.	99.	99.	[illegible]
19	9.0	99.	99.	99.	99.	99.	99.	99.	99.	99.	99.	99.	[illegible]
20	10.0	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	[illegible]
TOTAL OBS		3069	3069	3069	3069	3069	3069	3069	3069	3069	3069	3069	[illegible]

TABLE	MEAN	VARIANCE	C.OF V.	SKEWNESS	KURTOSIS
25	-0.053	5.109	42.397	5.341	77.949
26	0.129	10.061	24.506	4.125	78.239
27	-0.045	8.853	66.182	4.385	93.030
28	0.100	9.550	30.910	6.174	68.518
29	0.026	8.149	108.470	5.402	63.945
30	0.200	8.488	14.501	5.441	63.400
31	-0.071	6.962	37.388	5.343	65.975
32	-0.056	6.636	46.320	4.981	61.134
33	-0.131	8.215	21.807	3.040	50.709
34	-0.006	7.973	439.602	3.788	65.189
35	-0.106	8.917	28.219	5.369	61.054
36	-0.302	7.851	9.281	4.194	75.272

Table II-4. **Cumulative Probabilities for Period Errors Z for 1977-78 Base Year Forecasts With General Program Factors for Sample OC.H**

PERCENTAGES

		1	2	3	4	5	6	7	8	9	10	11	12
1	-9.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
2	-8.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
3	-7.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
4	-6.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
5	-5.0	0.	0.	1.	1.	0.	1.	0.	0.	0.	0.	0.	0.
6	-4.0	1.	1.	1.	1.	1.	1.	0.	0.	0.	0.	0.	0.
7	-3.0	2.	1.	2.	3.	2.	3.	0.	0.	0.	0.	0.	0.
8	-2.0	6.	5.	7.	4.	6.	8.	0.	0.	0.	0.	0.	0.
9	-1.0	25.	20.	23.	23.	23.	26.	0.	0.	0.	0.	0.	0.
10	0.	61.	57.	61.	60.	60.	62.	0.	0.	0.	0.	0.	0.
11	1.0	81.	78.	80.	83.	78.	79.	0.	0.	0.	0.	0.	0.
12	2.0	90.	87.	89.	91.	87.	88.	0.	0.	0.	0.	0.	0.
13	3.0	94.	93.	93.	94.	91.	93.	0.	0.	0.	0.	0.	0.
14	4.0	96.	95.	95.	96.	94.	95.	0.	0.	0.	0.	0.	0.
15	5.0	97.	97.	96.	97.	96.	96.	0.	0.	0.	0.	0.	0.
16	6.0	98.	98.	97.	98.	97.	97.	0.	0.	0.	0.	0.	0.
17	7.0	99.	98.	98.	98.	98.	98.	0.	0.	0.	0.	0.	0.
18	8.0	99.	99.	98.	98.	98.	98.	0.	0.	0.	0.	0.	0.
19	9.0	99.	99.	99.	99.	99.	98.	0.	0.	0.	0.	0.	0.
20	10.0	100.	100.	100.	100.	100.	100.	0.	0.	0.	0.	0.	0.
TOTAL OBS		2934	2934	2934	2934	2934	2934	0	0	0	0	0	0

TABLE	MEAN	VARIANCE	C.OF V.	SKEWNESS	KURTOSIS
37	0.020	5.573	117.334	2.336	75.153
38	0.257	6.263	9.726	6.008	76.853
39	0.159	8.734	18.591	5.777	65.792
40	-0.011	9.360	287.474	5.519	79.677
41	0.259	9.440	11.863	4.407	61.283
42	0.205	12.497	17.266	5.493	64.477

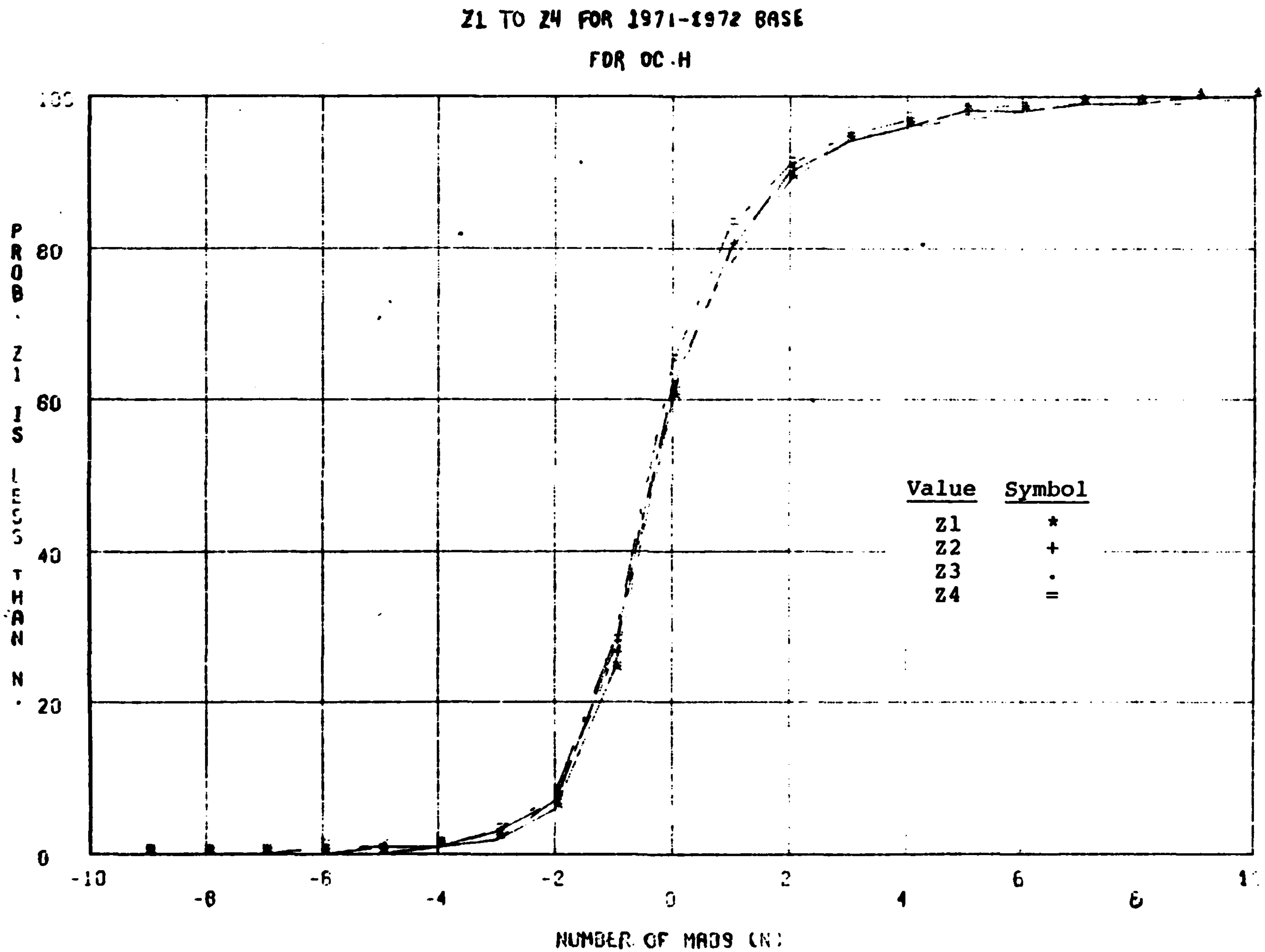


Figure II-14. Z1 to Z4 For 1971-1972 Base For OC.H.

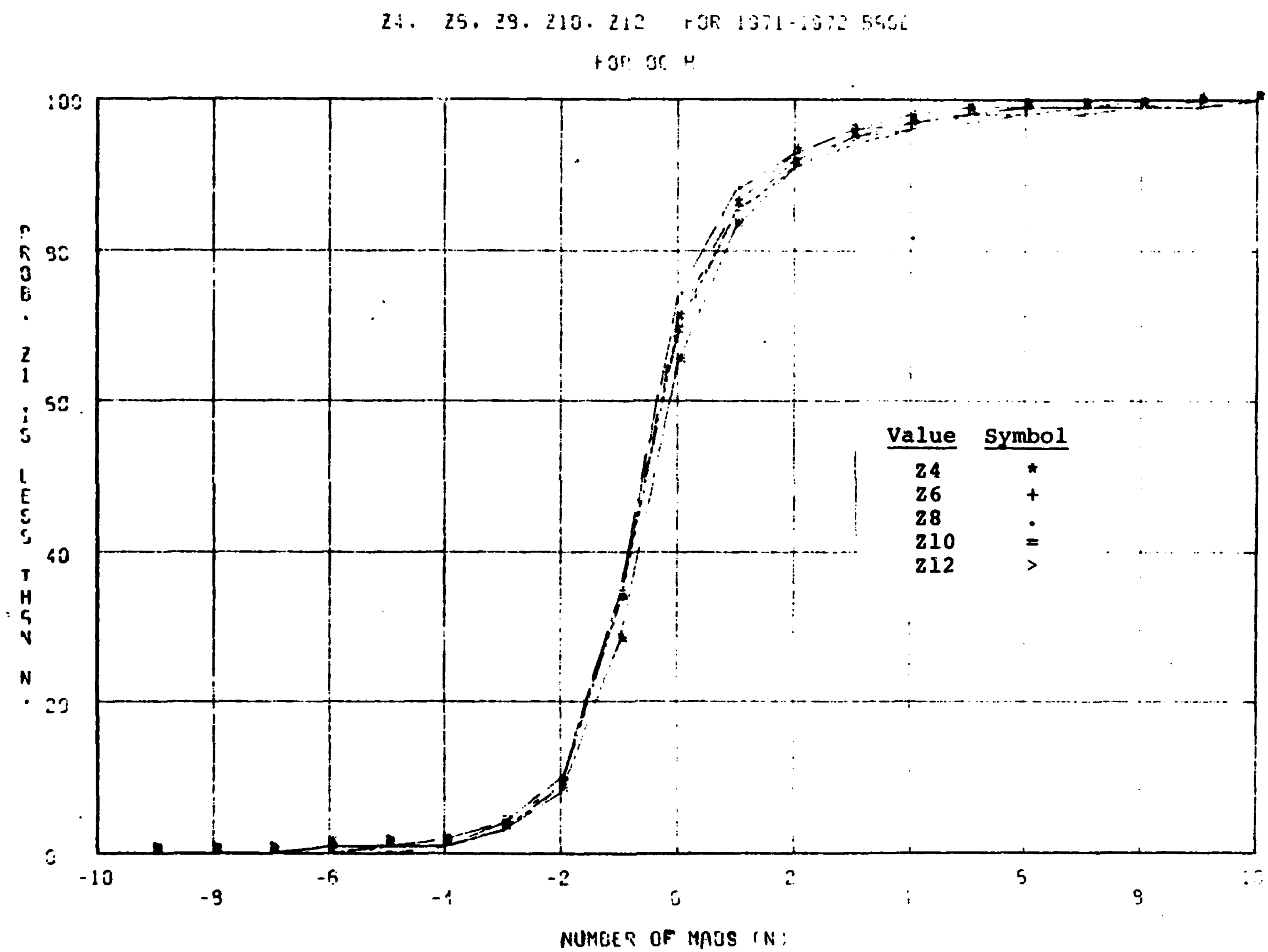


Figure II-15. Z4, Z6, Z8, Z10, Z12 for 1971-1972 Base For OC.H.

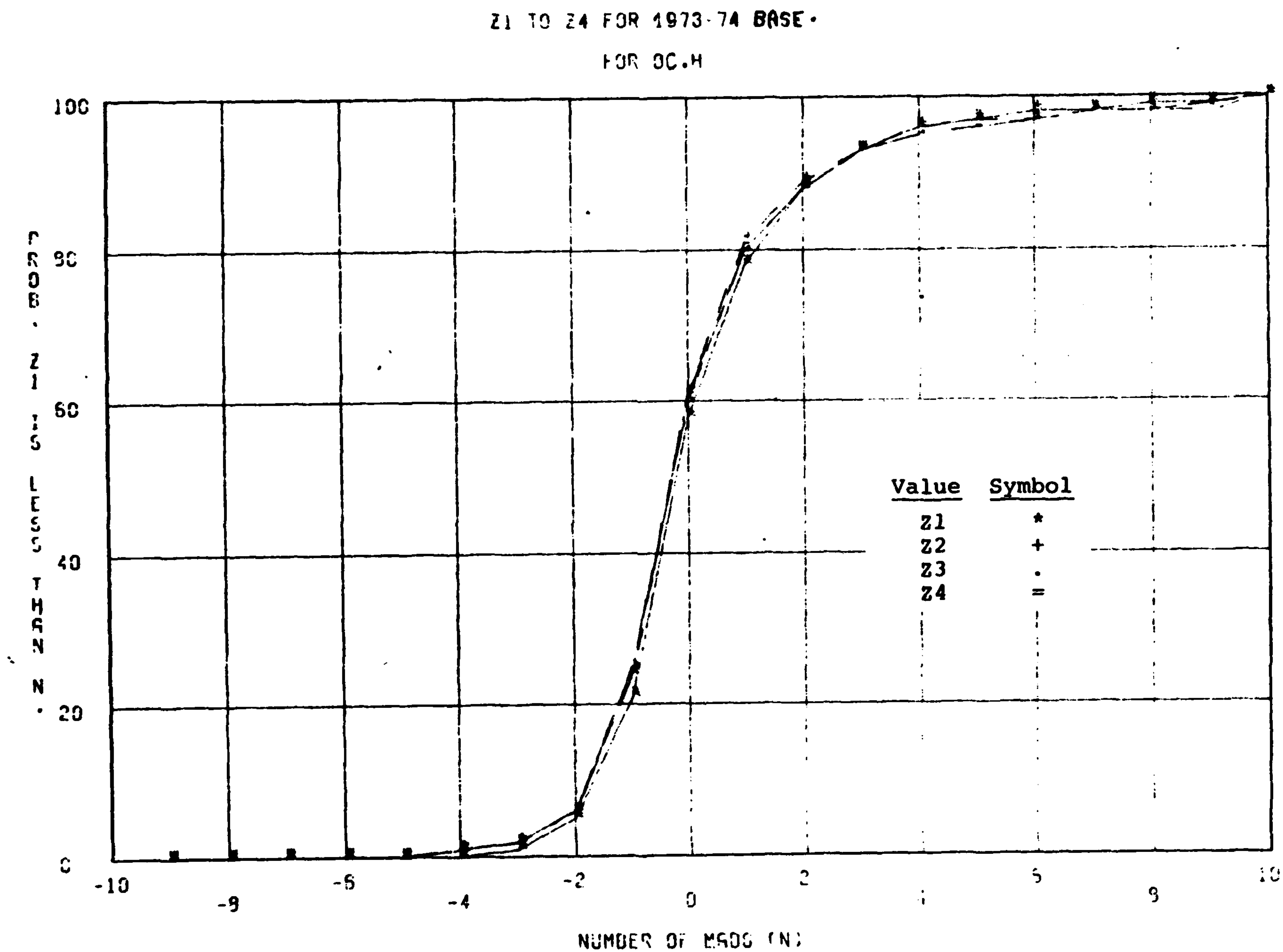


Figure II-16. Z1 to Z4 for 1973-1974 Base for OC.H.

#3
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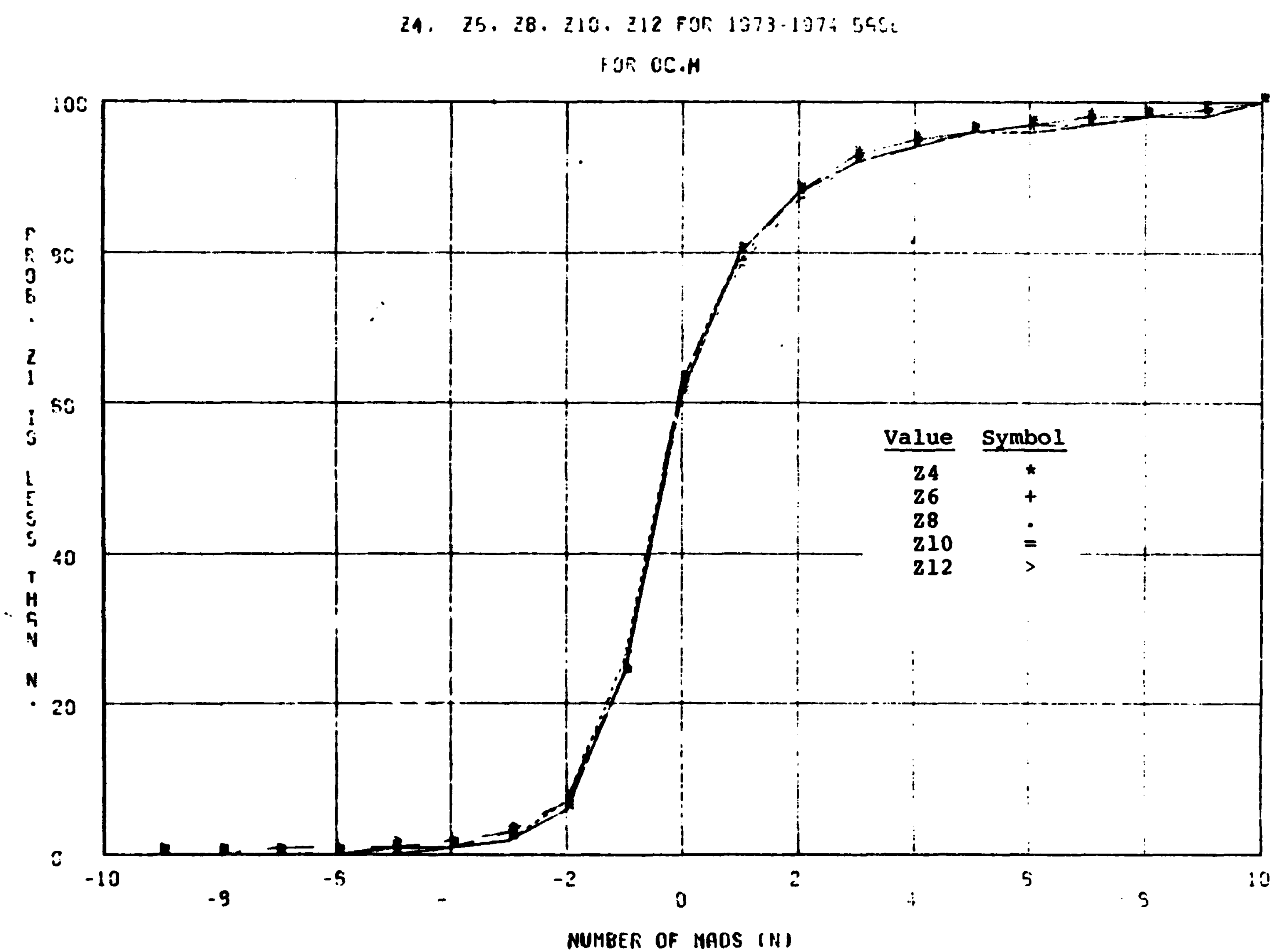

Figure II-17. Z4, Z6, Z8,Z10, Z12 For 1973-1974 Base For OC.H.

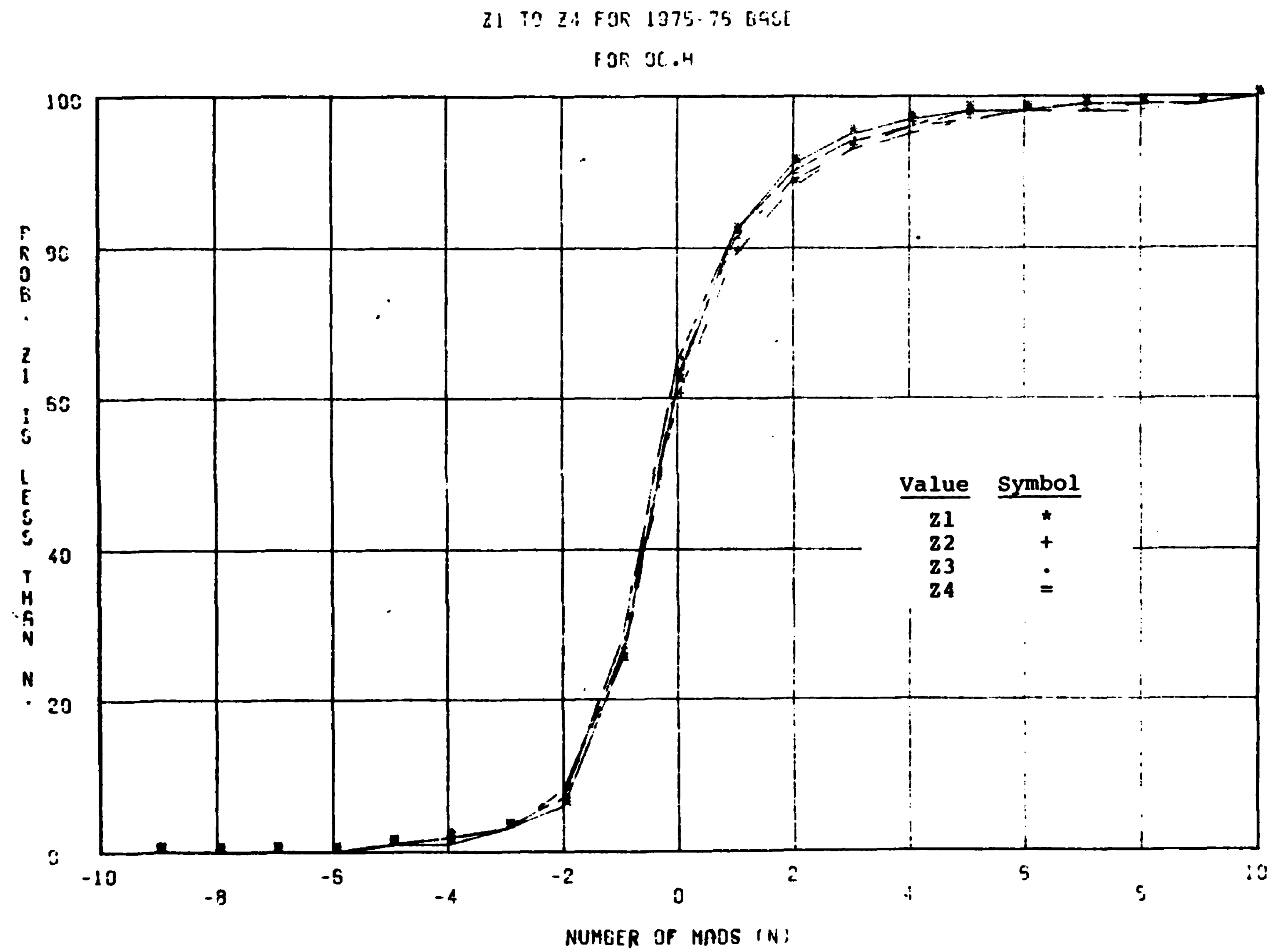

Figure II-18. Z1 to Z4 for 1975-1976 Base for OC.H.

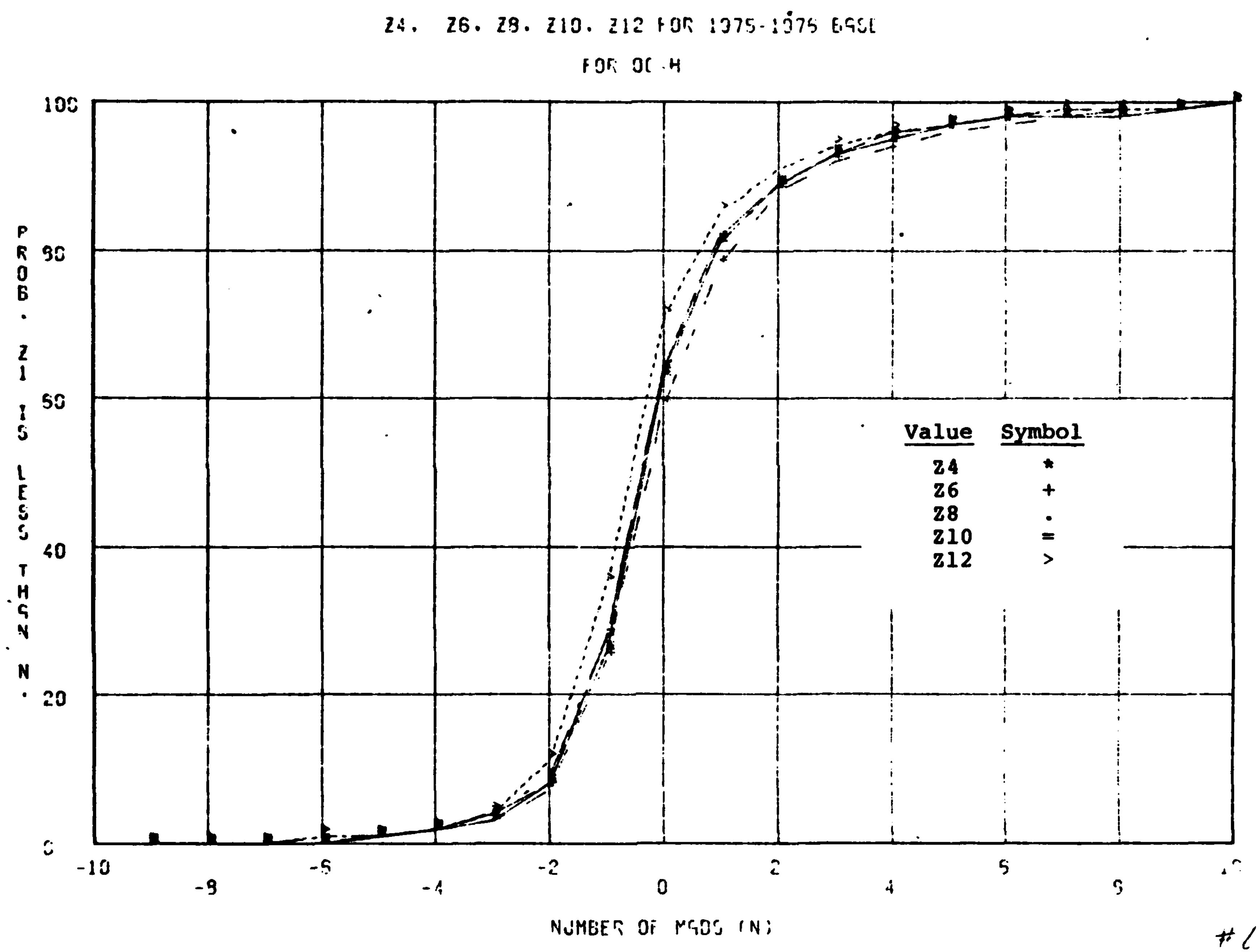

Figure II-19. Z4, Z6, Z8, Z10, Z12 for 1975-1976 Base for OC.H.

#1

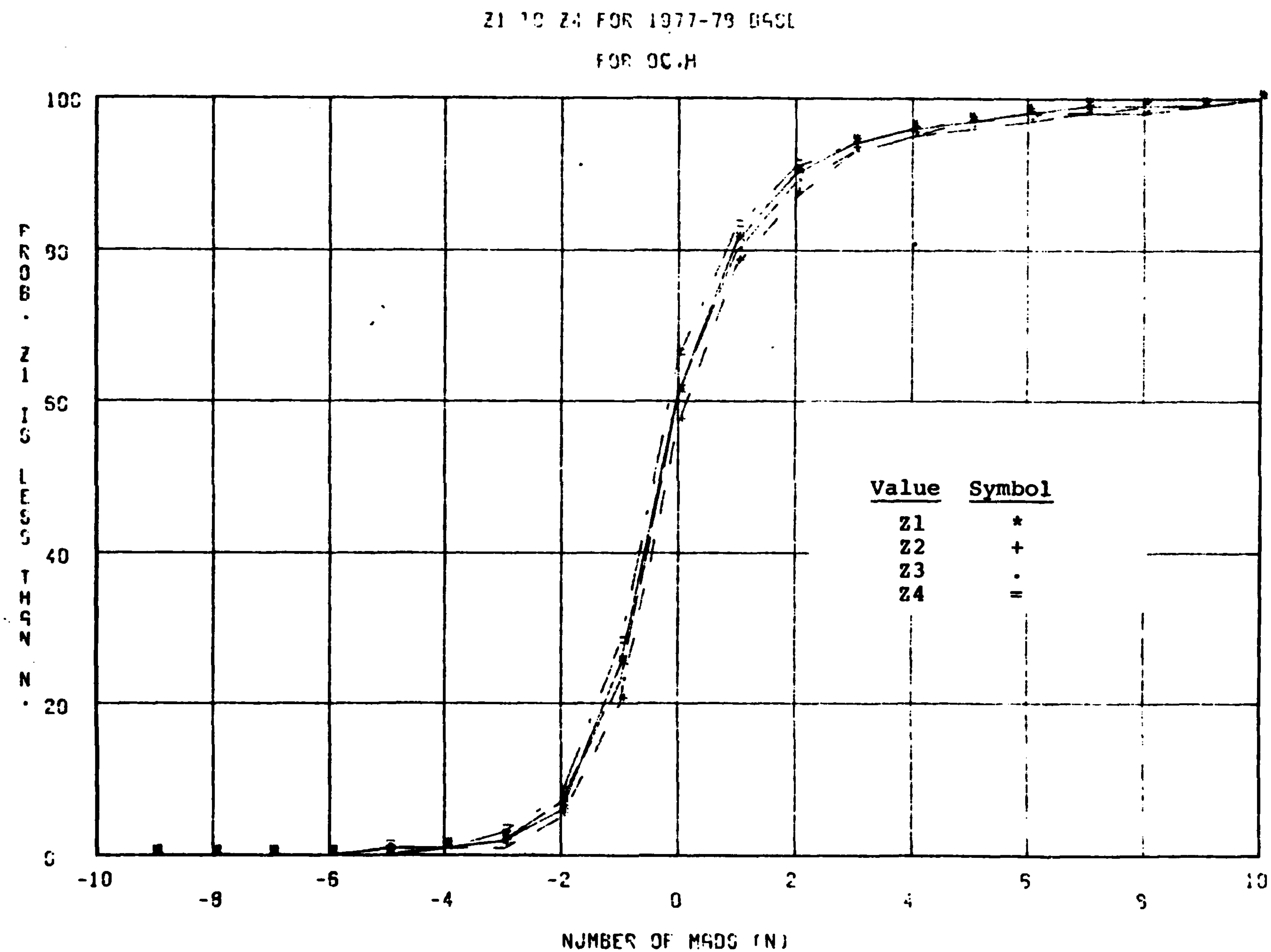

Figure II-20. Z1 to Z4 for 1977-1978 Base for OC.H.

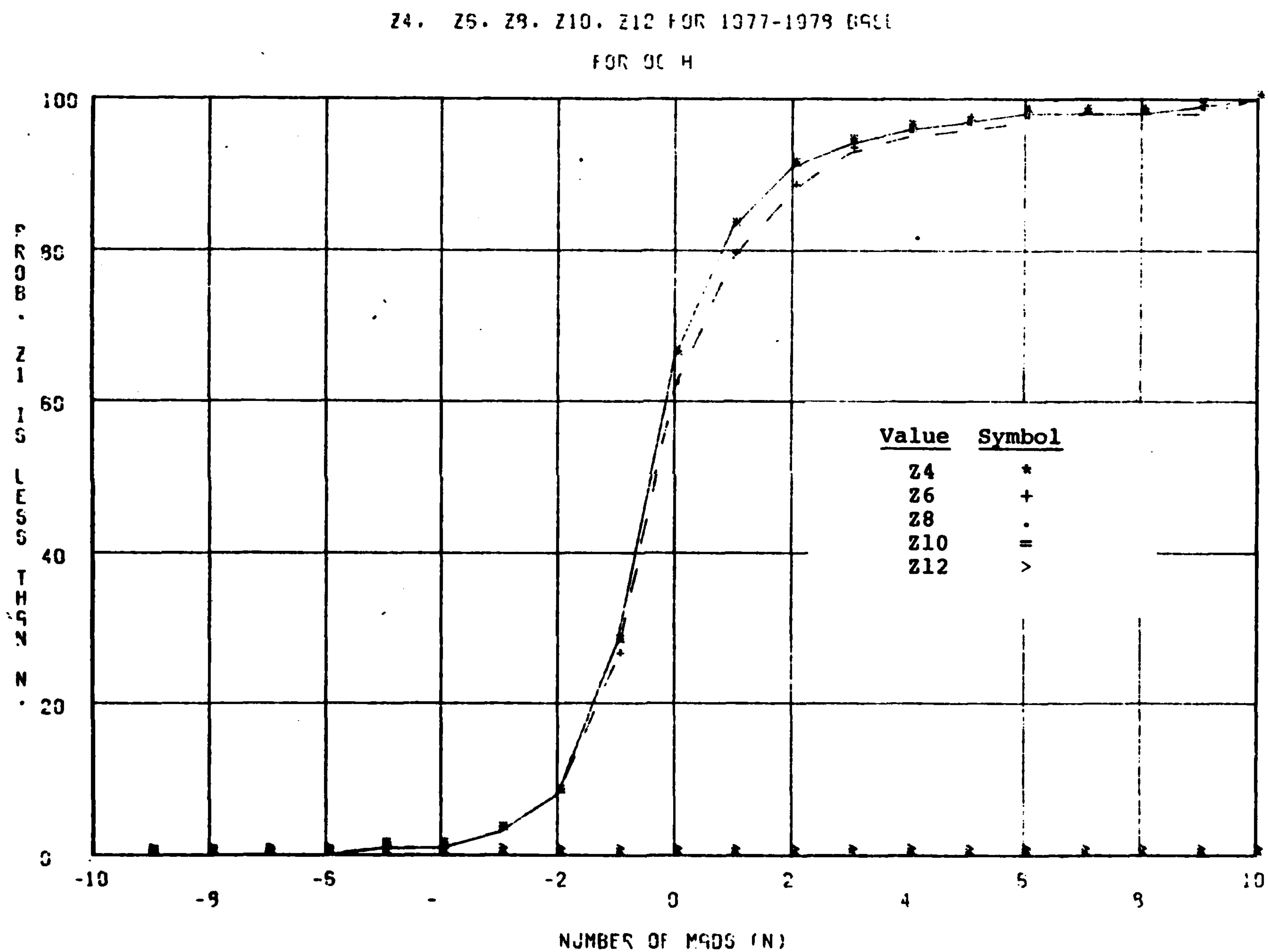

Figure II-21. Z4, Z6, Z8, Z10, Z12 for 1977-1978 Base for OC.H.

Let us now consider Table II-1. Column 1 of this table presents the Cumulative Distribution Function (CDF) associated with forecast errors Z1, i.e., the observed standardized error Zi when data for base year CY71-72 is used to forecast demands for the first quarter of CY73. Similarly, Column 2 of this table presents the CDF of forecast errors associated with Z2, i.e. errors in forecasting demands during the second quarter of CY73. Finally, column 12 denotes the CDF associated with Z12, the distribution of forecast errors when CY71-72 data is used to forecast demands twelve quarters into the future . Note that Samples OC.H contained a total of 3153 items. The mean, variance, coefficient of variation, skewness, and kurtosis statistics associated with each of these distributions is presented at the bottom of the table. Observe that the variance of each of these distributions is extremely high, that the distributions tend to be skewed, and that the kurtosis values are particularly large. In contrast, if the CDF of forecast errors were normally distributed, we would expect to see mean, standard deviation, skewness, and kurtosis values of 0, 1, 0, and 3, respectively.

Tables II-2 through II-4 present similar information for the CY73-74, CY75-76, and CY77-78 base year forecast, respectively.

Figures II-14 thru II-21 present graphs of the information presented in Tables II-1 through II-4. Figure II-14 presents plots of period errors Z1, Z2, Z3, and Z4 associated with CY71-72 base year forecasts for sample OC.H. As shown in the figure,

these CDFs appear to be identical. Similarly, Figure II-15 plots the cumulative distribution function for the period errors Z4, Z6, Z8, Z10, and Z12 for CY71-72 forecasts for sample OC.H. Note that the CDFs for these Zi appear to be very similar to the Z1 through Z4 curves. Figures II-16 through II-21 present similar plots associated with the other base year forecasts. Observe that there is very little difference among these curves across all base periods and lead times.

As noted above, similar information is presented in Appendix B for the OC.L, SM.H, and SM.L samples.

General Observations

After analyzing this information in detail, we believe that several general conclusions may be reached. First, within a given item sample and base period, the distribution of period errors Zi appear to be almost identical for all i. There is a minor tendency to over-forecast more as the forecast horizon increases, but this tendency appears slight. Second, for all items and samples, all of the curves are very similar. This is particularly true for Zi values that are greater than zero. Finally,the low demand item samples appear to have a slightly higher probability of over-forecasting than for the higher activity samples. In the next section, we perform studies to identify the impact of differing demand rates upon the standardized error distributions.

Section III

Analysis of Forecast Errors for Demand in a Leadtime for High Activity Items

In Section II, we observed significant differences among item groups based on average dollar demands in a base period. This is not surprising. Anaylsis of item demand histories shows that many low demand D062 items have many periods of zero demands, but that when demands do occur, they are often of large magnitude relative to the average usage rate. On the other hand, high activity D062 items appear to be less erratic though still highly variable. As a result, we conducted two basic categories of studies: (1) analyses of items with relatively high usage rates of three units per quarter or more, and (2) studies of items with lower levels of activity. In this Section, we consider the high usage rate items, while Section IV presents our results for low activity items.

Definition of Standarized Forecast Errors

We are interested in identifying the characteristics of errors in forecasting the total units demanded in a given number of time periods. From probability theory, it is well known that if demands per period are identically distributed random variables, then the expected value of a sum of t of these variables equals t times the expected value of demand in a single period. Similarly, if the variables are independent, the variance of the

sum is equal to t times the variance of demand in an individual period. In symbols, let X_i denote the demand in period i, and suppose X_i has mean u and standard deviation s_x for all periods i. Let D denote the total demand observed in t periods; i.e.

(1) $D = X_1 + X_2 + \ldots + X_t$

Since the X_i are independent and identically distributed,

(2) $E(D) = t \cdot E(X_i) = t\,u$

and

(3) $\mathrm{Var}(D) = t \cdot \mathrm{Var}(X_i) = t\,s_x^{\,2}$

Hence, the standard deviation of D is given by

$$s_d = s_x \sqrt{t}$$

From the studies discussed in Section II, we know that period errors are correlated. Consequently we do not necessarily expect relationship (3) above to hold. Nevertheless, we found it useful to standardize forecast errors in a manner similar to that employed when demands are uncorrelated. Specifically, let R denote the forecasted quarterly demand rate associated with each future period, and let MAD denote the Mean Absolute Deviation associated with the observed demand in the base period. Then we define the standardized forecast errors for demand in the lead time as:

$$CZ_t = \frac{\begin{bmatrix}\text{Total Demand Observed} \\ \text{in t quarters}\end{bmatrix} - R \cdot t}{\mathrm{MAD}\sqrt{t}}$$

or,

$$CZ_t = \frac{D - R\,t}{MAD\ \sqrt{t}}$$

If forecast errors are in fact normally distributed — as assumed in the current D062 saftey level calculations -- then the observed distribution for the standardized error CZ should be normally distributed with a mean of zero and a standard deviation of 1.25 (since for the standard normal distribution the MAD is approximately equal to .8s).

Demand Class Numbering Comentions

As noted above, we observed significant differences in the forecast error characteristics among differing demand classes. Consequently, we assigned individual item forecasts to specific demand classes. Table III-1 presents the assignment of Histogram Numbers to specific forecast demand rate categories. For example, histograms 1 thru 6 were associated with items which had demand forecasts in the range of 3 to 6 units per quarter. Histogram 1 developed the cumulative distribution function (CDF) associated with CZ_1, i.e. the distribution of errors in forecasting demands for one quarter into the future. Histogram two, on the other hand, tabulated the histogram of standardized errors CZ_2 associated with errors in forecasting the total demand in quarters 1 and 2 combined. Similarly, tables 3, 4, 5 and 6 tabulated the CDFs associated with standardized errors in forecasting the total demands in 3, 4, 5, and 6 quarters, respectively. In all of these cases, the forecast demand rate was in the 3 to 6 units range. Similarily, Histograms 7 thru 12 were assigned to items with forecast demand rates in the range of 6 to 10 units per quarter. For items in this demand rate category, table 7 tabulated the CDF of standardized errors CZ_1 for forecasts one quarter into the future, Histogram 8 tabulated

Table III-I

Assignment of Histogram Numbers in Analysis

of CZ_1/k Distributions

Histogram Numbers	Forecast Demand per Quarter (Units)
1-6	3-6
7-12	6-10
13-18	10-31
19-24	31-100
25-30	100-310
31-36	310-1000
37-42	1000-3100

the errors in forecasting total demands for two future quarters, and so on. Other histogram number assignments and the corresponding forecast demand rate categories are illustrated in Table III-1.

Let $k = MAD\sqrt{t}$. We calculated the statistic CZ/k for each of the four item samples SM.L, SM.H, OC.L, and OC.H, and tabulated our results using the Histogram numbering scheme shown in Table III-1. Our results for Sample SM.L are presented in Table III-2. For example, consider the second and third columns of the table. Column 2 presents values for the CZ/k statistic, while column 3 presents percentage counts for histogram 1, i.e. counts for forecasts in the 3-6 units/qtr range. As shown in these columns, CZ/k values were less than or equal to -1 in 26% of the forecasts in this range, 76% of the CZ/k values were less than or equal to 0, and 86% of the CZ/k values were less than or equal to 1. Other percentage values for CZ/k in the range from -9 to 10 may be read from column 3. Other columns of Table III-2 present similar statistics for each of the Histograms defined in Table III-1.

Since a picture is often worth one thousand words, let us now consider the shapes of these distributions.

Table III-2.

Observed CZ/k CDFs for Sample SM.L.

CZLTSM.L

TABLES 1 - 12

1	-9.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
2	-8.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
3	-7.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.
4	-6.	0.	0.	0.	0.	1.	1.	0.	0.	0.	1.	1.	1.
5	-5.	0.	0.	1.	1.	1.	1.	0.	0.	1.	2.	2.	3.
6	-4.	0.	1.	1.	2.	3.	4.	0.	1.	2.	3.	4.	5.
7	-3.	1.	3.	4.	6.	9.	12.	2.	3.	5.	9.	11.	13.
8	-2.	5.	11.	16.	21.	26.	31.	6.	12.	19.	24.	29.	32.
9	-1.	29.	46.	52.	55.	57.	59.	33.	44.	51.	55.	58.	60.
10	0.	76.	74.	73.	74.	75.	76.	73.	75.	74.	75.	77.	79.
11	1.	86.	85.	85.	84.	85.	85.	86.	87.	85.	86.	86.	86.
12	2.	92.	92.	91.	90.	90.	90.	92.	93.	91.	91.	90.	90.
13	3.	95.	95.	94.	94.	94.	93.	95.	96.	95.	94.	93.	92.
14	4.	97.	97.	97.	96.	95.	95.	97.	98.	96.	96.	95.	96.
15	5.	98.	98.	98.	97.	97.	97.	98.	98.	97.	97.	97.	97.
16	6.	99.	99.	99.	98.	98.	98.	99.	98.	98.	98.	98.	98.
17	7.	99.	99.	99.	99.	99.	98.	99.	99.	99.	99.	98.	98.
18	8.	99.	99.	99.	99.	99.	99.	99.	99.	99.	99.	99.	99.
19	9.	99.	100.	99.	99.	99.	99.	99.	99.	99.	99.	99.	99.
20	10.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.

CZLTSM.L

TABLES 13 - 24

1	-9.	0.	0.	0.	0.	1.	1.	0.	0.	0.	0.	0.	1.
2	-8.	0.	0.	0.	0.	1.	1.	0.	0.	0.	0.	1.	1.
3	-7.	0.	0.	1.	1.	1.	1.	0.	0.	1.	1.	1.	1.
4	-6.	0.	1.	1.	1.	2.	2.	0.	1.	1.	2.	2.	2.
5	-5.	0.	1.	2.	2.	3.	4.	1.	2.	2.	2.	3.	4.
6	-4.	1.	2.	3.	4.	6.	8.	1.	2.	4.	5.	6.	8.
7	-3.	2.	4.	7.	10.	13.	17.	2.	5.	9.	13.	15.	21.
8	-2.	7.	15.	23.	28.	34.	39.	8.	15.	23.	29.	36.	41.
9	-1.	35.	51.	55.	57.	60.	63.	37.	50.	56.	57.	61.	64.
10	0.	74.	73.	73.	74.	76.	77.	76.	71.	74.	75.	77.	76.
11	1.	86.	85.	86.	84.	84.	84.	85.	89.	87.	85.	86.	85.
12	2.	92.	91.	91.	90.	90.	89.	93.	94.	93.	90.	90.	88.
13	3.	96.	95.	95.	93.	94.	93.	96.	96.	95.	93.	93.	92.
14	4.	97.	97.	97.	96.	96.	95.	97.	97.	97.	95.	95.	94.
15	5.	98.	98.	98.	97.	97.	96.	99.	97.	97.	96.	96.	96.
16	6.	99.	98.	98.	98.	98.	97.	99.	98.	98.	97.	97.	97.
17	7.	99.	99.	99.	98.	98.	98.	99.	98.	98.	98.	98.	98.
18	8.	99.	99.	99.	99.	99.	98.	99.	98.	98.	98.	99.	98.
19	9.	99.	99.	99.	99.	99.	99.	99.	99.	99.	99.	99.	98.
20	10.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.

CZLTSM.L

TABLES 25 - 36

1	-9.	0.	0.	0.	0.	0.	0.	0.	5.	5.	5.	5.	5.
2	-8.	0.	0.	0.	0.	0.	0.	0.	5.	5.	5.	5.	5.
3	-7.	0.	0.	0.	0.	1.	1.	0.	5.	5.	5.	5.	9.
4	-6.	0.	0.	0.	1.	1.	1.	5.	5.	5.	5.	5.	9.
5	-5.	0.	0.	1.	1.	3.	6.	5.	5.	5.	9.	9.	9.
6	-4.	1.	2.	5.	9.	13.	15.	5.	5.	5.	9.	9.	14.
7	-3.	2.	12.	18.	23.	23.	29.	5.	5.	9.	14.	18.	23.
8	-2.	14.	28.	35.	36.	45.	46.	23.	23.	36.	32.	36.	45.
9	-1.	46.	53.	60.	57.	64.	65.	41.	55.	50.	45.	45.	45.
10	0.	72.	75.	75.	74.	76.	79.	59.	68.	73.	73.	73.	73.
11	1.	86.	88.	88.	83.	84.	85.	77.	82.	86.	82.	82.	86.
12	2.	96.	91.	91.	91.	89.	88.	86.	86.	86.	91.	95.	91.
13	3.	98.	96.	93.	92.	91.	91.	86.	86.	91.	95.	95.	95.
14	4.	98.	98.	96.	96.	95.	95.	86.	95.	100.	100.	95.	95.
15	5.	99.	100.	97.	97.	97.	96.	91.	100.	100.	100.	100.	95.
16	6.	100.	100.	99.	97.	97.	97.	95.	100.	100.	100.	100.	100.
17	7.	100.	100.	100.	97.	97.	97.	95.	100.	100.	100.	100.	100.
18	8.	100.	100.	100.	99.	98.	98.	100.	100.	100.	100.	100.	100.
19	9.	100.	100.	100.	100.	99.	98.	100.	100.	100.	100.	100.	100.
20	10.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.

CZLTSM.L

TABLES 37 - 48

1	-9.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
2	-8.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
3	-7.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
4	-6.	0.	0.	0.	0.	0.	50.	0.	0.	0.	0.	0.	0.
5	-5.	0.	0.	50.	0.	50.	50.	0.	0.	0.	0.	0.	0.
6	-4.	0.	0.	50.	50.	50.	50.	0.	0.	0.	0.	0.	0.
7	-3.	0.	0.	50.	50.	100.	100.	0.	0.	0.	0.	0.	0.
8	-2.	0.	100.	100.	100.	100.	100.	0.	0.	0.	0.	0.	0.
9	-1.	50.	100.	100.	100.	100.	100.	0.	0.	0.	0.	0.	0.
10	0.	100.	100.	100.	100.	100.	100.	0.	0.	0.	0.	0.	0.
11	1.	100.	100.	100.	100.	100.	100.	0.	0.	0.	0.	0.	0.
12	2.	100.	100.	100.	100.	100.	100.	0.	0.	0.	0.	0.	0.
13	3.	100.	100.	100.	100.	100.	100.	0.	0.	0.	0.	0.	0.
14	4.	100.	100.	100.	100.	100.	100.	0.	0.	0.	0.	0.	0.
15	5.	100.	100.	100.	100.	100.	100.	0.	0.	0.	0.	0.	0.
16	6.	100.	100.	100.	100.	100.	100.	0.	0.	0.	0.	0.	0.
17	7.	100.	100.	100.	100.	100.	100.	0.	0.	0.	0.	0.	0.
18	8.	100.	100.	100.	100.	100.	100.	0.	0.	0.	0.	0.	0.
19	9.	100.	100.	100.	100.	100.	100.	0.	0.	0.	0.	0.	0.
20	10.	100.	100.	100.	100.	100.	100.	0.	0.	0.	0.	0.	0.

Plots of CZ/K

Let us now consider Figure III-1. The curves shown on the left hand side of this figure denote the cumulative Distribution Functions (CDF) of CZ/K associated with items having a forecast demand in the range of three to six units per quarter, while the plots on the right hand side of this figure denotes analytical approximations to this curve. As shown in the figure, for values of N greater than zero, the observed CDFs are almost identical for all six lead times; however, significant differences among the curves exist for negative N values. Figure III-1B presents three approximations to the CDFs presented in Figure III-1A. First, the solid line denotes the average for all six of the curves presented in Figure III-1A. The large dashed line presented in Figure III-1B presents the cummulative distribution function associated with the normal probability distribution. For the normal distribution the fiftieth percentile occurs when the random variable N equals zero, and the 99.99 percentile occurs for N equals to 3.62 standard deviations or (3.62) (1.25)= 4.52 MADS. Finally, the third curve presented in Figure II-1B is extremely difficult to see, because it lies almost on top of the average (solid line) curve. This third curve--represented by a line of small dashes--is the curve of a an exponential function which passes through the average curve at the point where N equals zero and N equals five. Note that this exponential approximation is an excellent fit to the observed data.

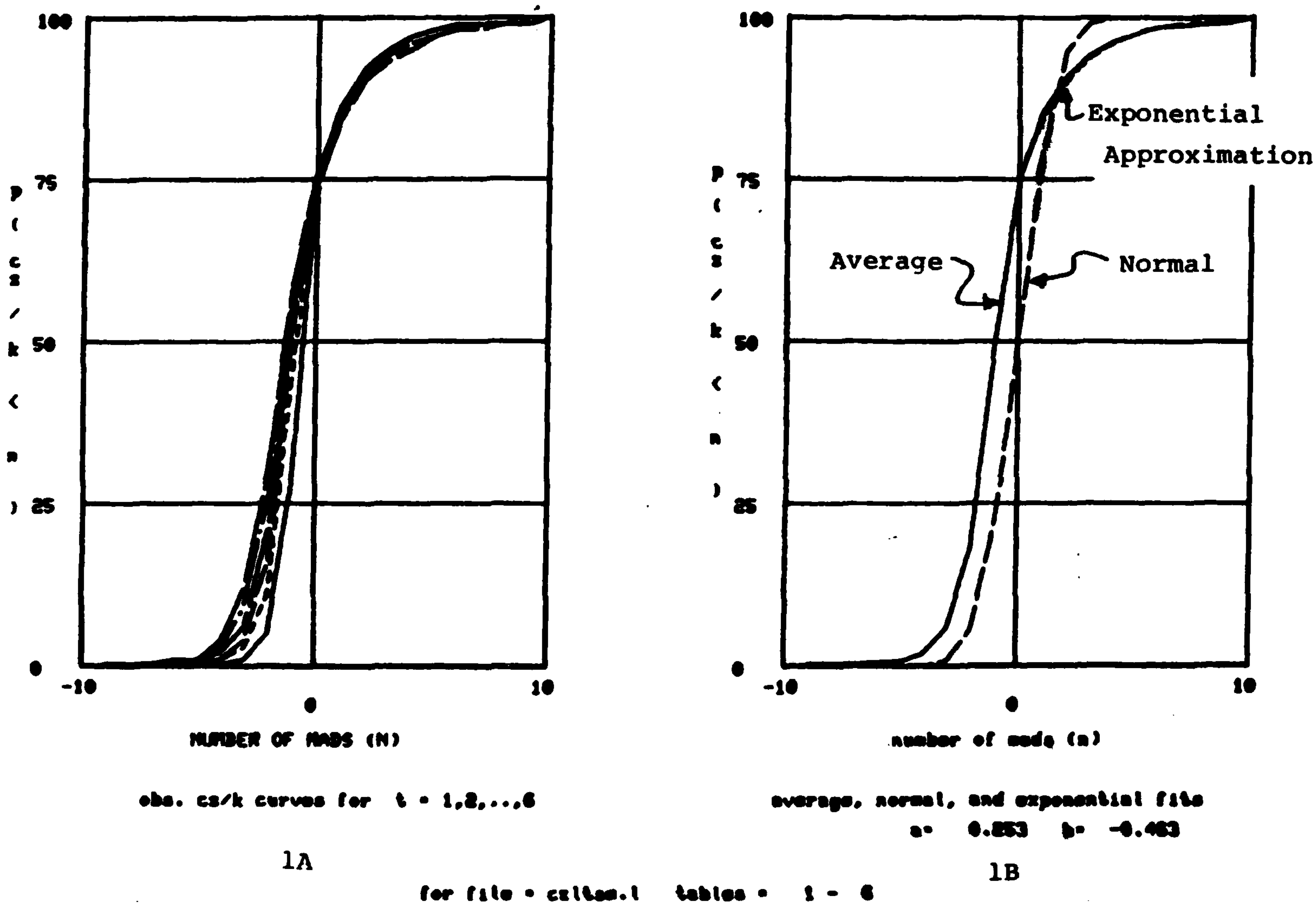

Figure III-1. CZ/k Curves for SM.L Forecasts of 3-6 Units per Quarter.

Figures III-2 thru III-6 presents results similar to Figure III-1 for demand classes in which the forecast is in the range of 6-10, 10-31, 31-100, 100-310, and 310-1000 units per quarter, respectively. Each of these curves present individual CDFs associated with the CZ/K statistic for each of the six different lead times. In addition, average, normal, and exponential approximations to each of these curves are also presented. Note that all of these curves are very similiar. In no case does the average CDF appear to be normally distributed. On the other hand, the exponential approximation provides an excellent fit to the upper tail of the observed cummulative distribution function in all of these figures.

If each of the curves presented in Figures III-1 thru III-6 are compared across demand rate groups, we observed that all of the curves are very similiar. For example, Table III-3 presents a comparison of the "average" curves associated with each demand rate group, and displays the maximum differences among these curves. As shown in the table, there is practically no difference between the curves for N values greater than zero. However, differences as large as 7% exist for N values less than zero. Fortunately, in computing safety levels, our primary interest is for N values which are zero or greater. Consequently, for safety level calculation purposes, it is useful for us to compute an average upper tail curve for this class items.

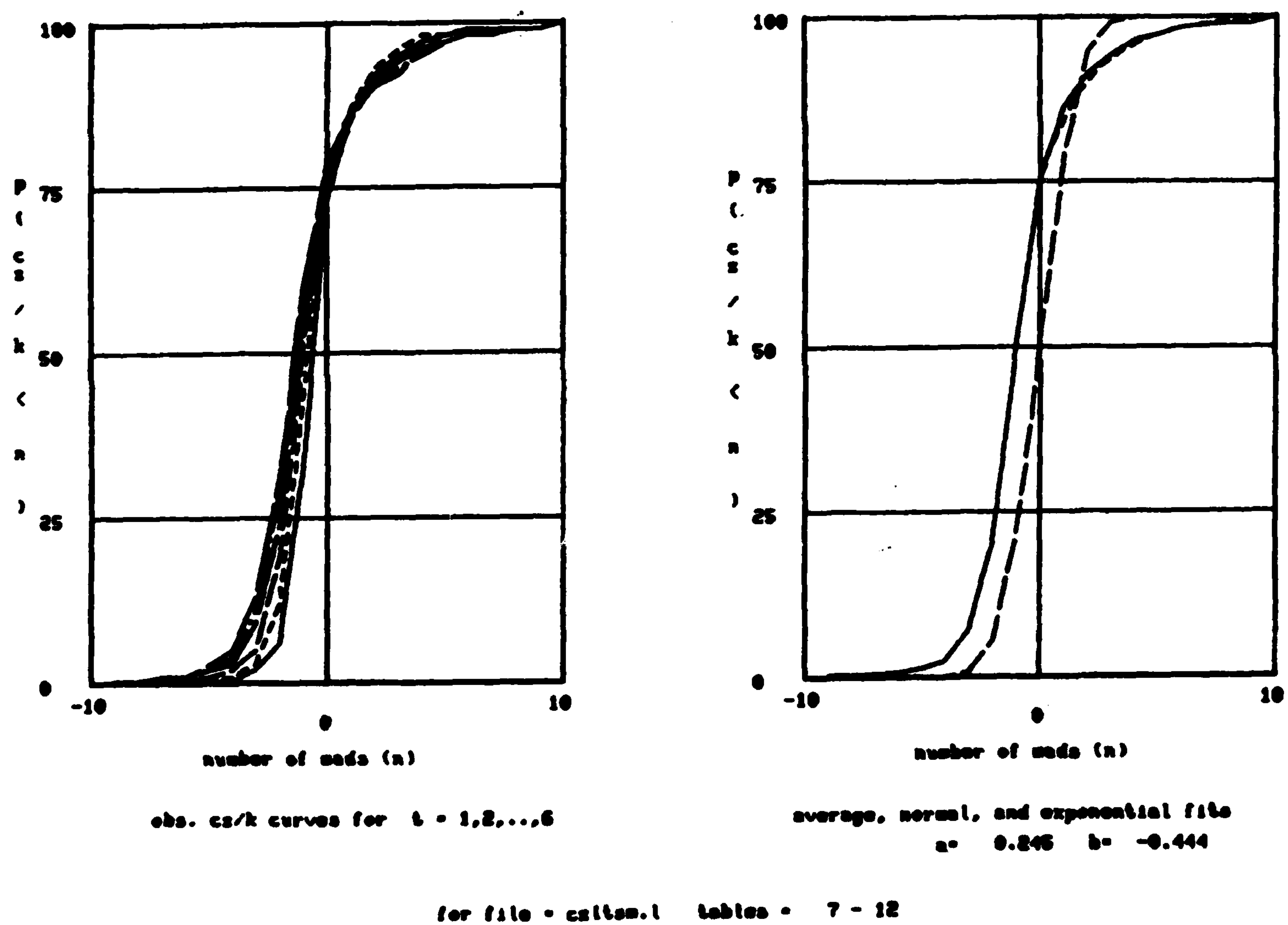

Figure III-2. CZ/k Curves for SM.L Forecasts of 6-10 Units per Quarter.

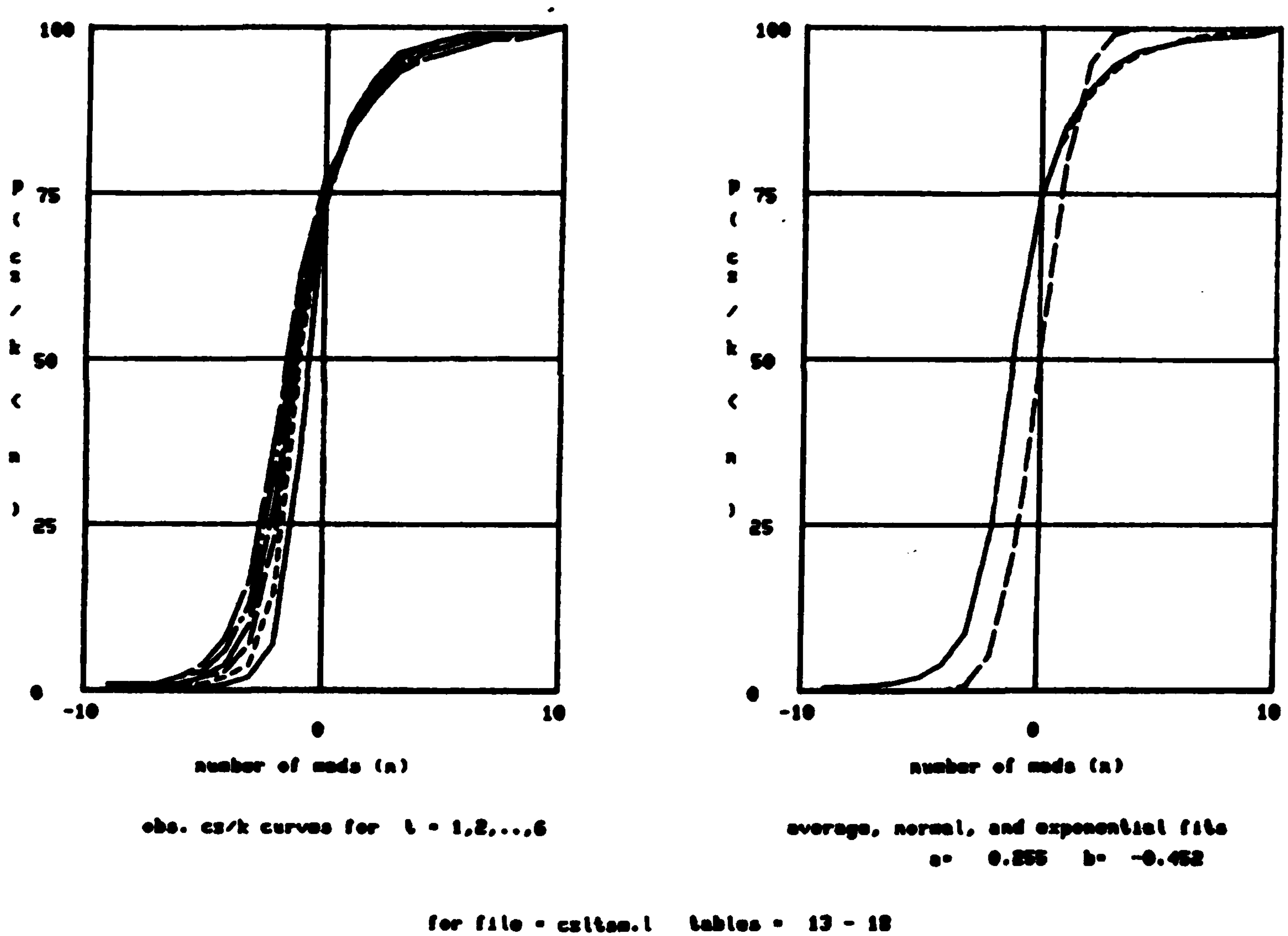

Figure III-3. CZ/k Curves for SM.L Forecasts of 10-31 Units per Quarter.

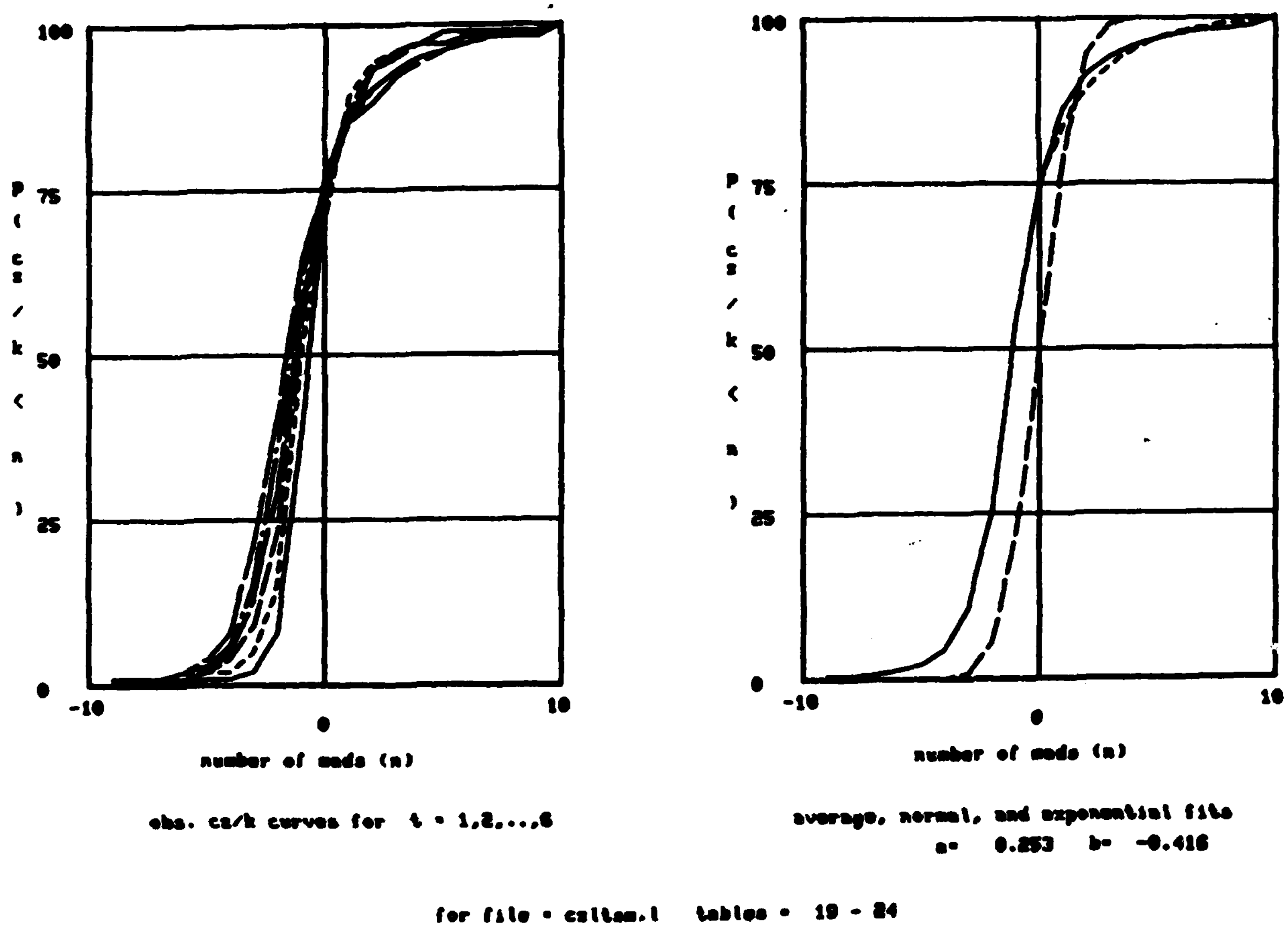

Figure III-4. CZ/k Curves for SM.L Forecasts of 31-100 Units per Quarter.

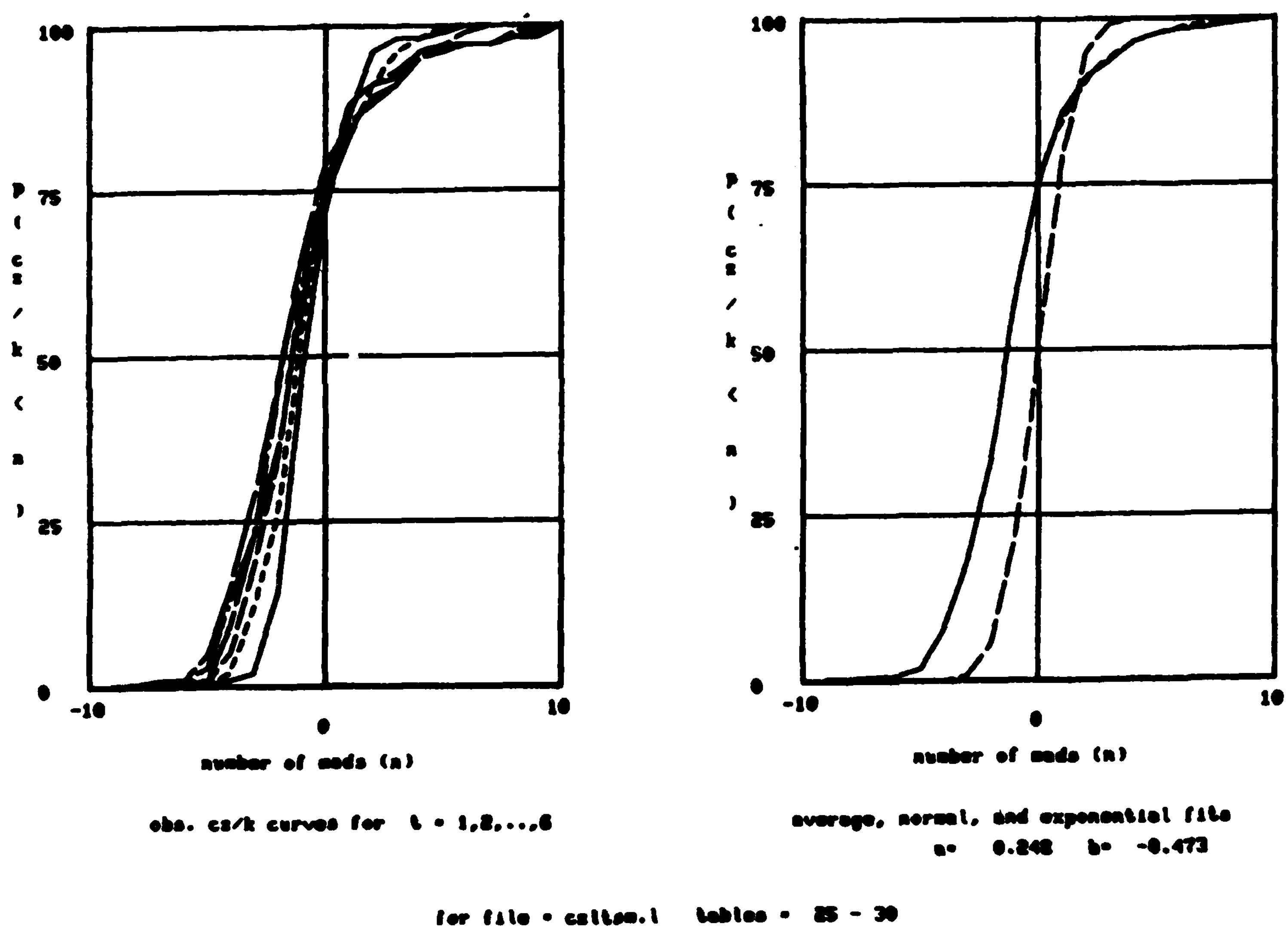

Figure III-5. CZ/k Curves for SM.L Forecasts of 100-310 Units per Quarter.

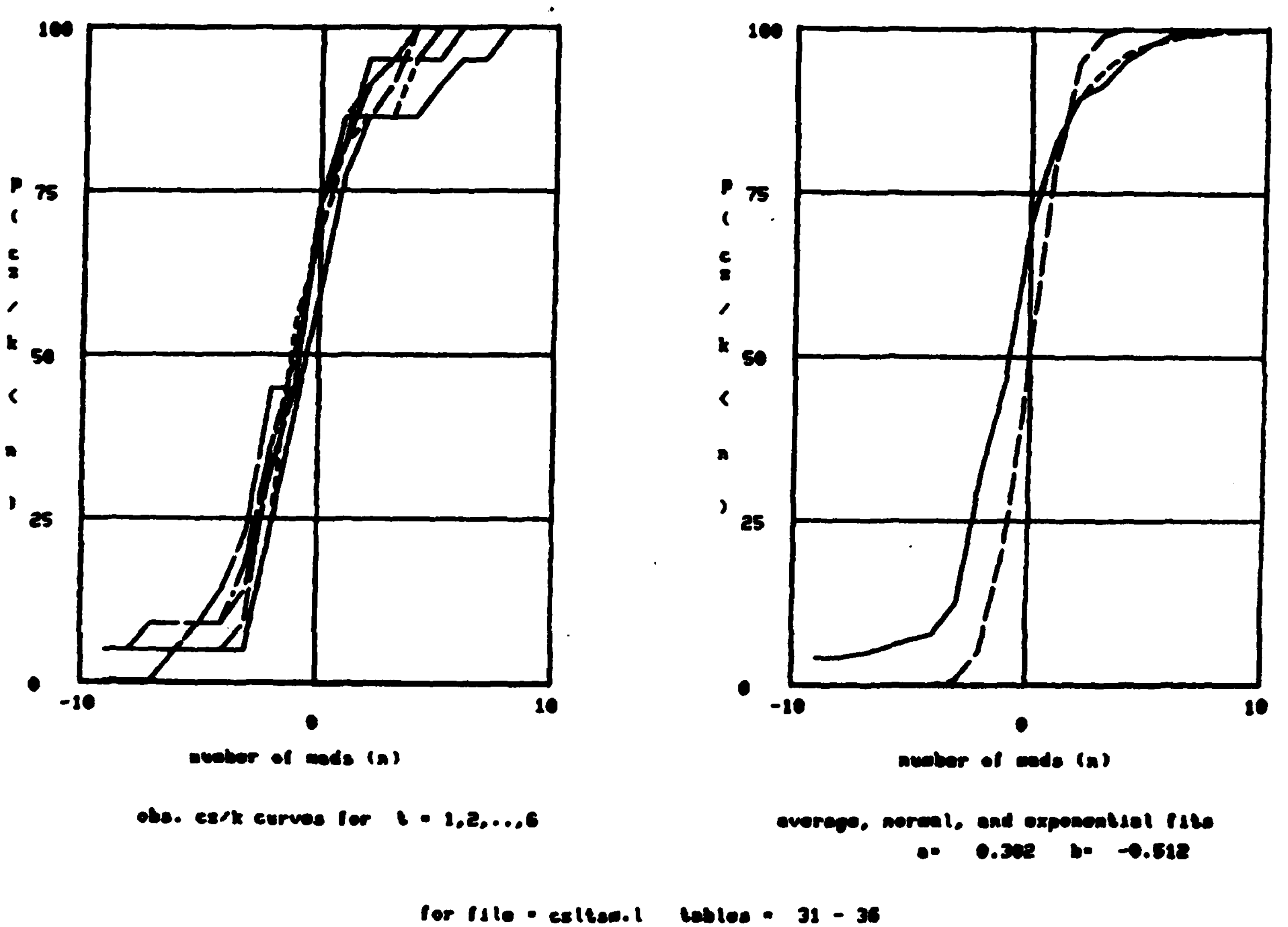

Figure III-6. CZ/k Curves for SM.L Forecasts of 310-1000 Units per Quarter.

TABLE III-3

COMPARISON OF AVERAGE CURVES OF P [cz/k ≤ z]
FOR SAMPLE SMIL

P [cz/k ≤ z] by Units/Qtr Category

z	3-6	6-10	10-31	31-100	MAXIMUM DIFFERENCE
-9	0	0	0	0	0
-8	0	0	0	0	0
-7	0	0	1	1	1
-6	0	1	1	1	1
-5	1	1	2	2	1
-4	2	3	4	4	2
-3	6	7	9	11	5
-2	18	20	24	25	7
-1	50	50	54	54	4
0	75	76	75	75	1
1	85	86	85	86	1
2	91	91	91	91	0
3	94	94	94	94	0
4	96	96	96	96	0
5	98	97	97	97	1
6	99	98	98	98	1
7	99	99	99	98	1
8	99	99	99	98	1
9	99	99	99	99	0
10	100	100	100	100	0
NO. OF OBSERVATIONS	2013	1045	1235	394	

NOTE: The Critical Kolomogrov - Smirnoff Statistic for N = 2000 and 95% confidence is 3%

Comparisons with Other ALC Samples

In the above paragraphs, we presented data associated with the SM.L sample. However, similiar findings apply to the other three samples used in this study. The detailed plots associated with each of these other samples are presented in Appendix C. In all cases, we observe that an exponential function is an excellent approximation to the average curves for N values greater than zero. Similiarly, we also find that a single curve provides a good approximation to the exponential tail across all demand rate groups. Unfortunately, it appears that each item sample is charaterized by its own exponential tail. To see this, let us consider Table III-4 and Figure III-8. Table III-4 presents the exponential tail coefficient estimates associated with all samples, while Figure III-8 plots each of these coefficient estimates. As shown in the table, the coefficient estimates are very similiar within an item grouping, but differ significantly from one item group to another.

Additional Characteristics of Forecast Errors

Figures III-9 thru III-11 present several other interesting charateristics of the distribution of forecast errors. Figure III-9 plots the mean value of CZ/K by demand catagory for the OC.H sample. Let us consider the six points on the left hand side of this figure. These points represent the Mean forecast of

Table III-4

Exponential Coefficient Estimates by Demand Class and Group

Model: F (z) = 1 - A exp (Bz)

Forecast Demand/QTR	Histogram	CZLTOC.L N	CZLTOC.L A	CZLTOC.L B	CZLTOC.H N	CZLTOC.H A	CZLTOC.H B
3-6	1-6	3401	.302	-.381	1184	.412	-.409
6-10	7-12	2070	.292	-.415	1172	.407	-.433
10-31	13-18	2806	.307	-.476	3000	.398	-.496
31-100	19-24	1204	.313	-.481	2619	.387	-.535
16-310	25-30	300	.300	-.599	1274	.392	-.483
31-1000	31-36	75	.310	-.629	565	.397	-.467
1000-3100	37-42	14	.262	-.402	237	.422	-.471
3100-10000	43-48	2	-	-	120	.317	-.536
WEIGHTED AVERAGE		9876	.303	-.436	10171	.397	-.485

Forecast Demand/QTR	Histogram	CZLTSM.L N	CZLTSM.L A	CZLTSM.L B	CZLTSM.H N	CZLTSM.H A	CZLTSM.H B
3-6	1-6	2013	253	-.463	360	.313	-.469
6-10	7-12	1045	.245	-.444	244	.345	-.439
10-31	12-18	1235	.255	-.452	505	.310	-.566
31-100	19-24	394	.253	-.416	203	.327	-.640
100-310	25-30	92	.245	-.473	59	.273	-.743
310-1000	31-36	22	.302	-.512	21	.372	-.299
	36-42	2	-	-	4	-	-
1000-3100	37-42	2	-	-	4	-	-
WEIGHTED AVERAGE		4801	.252	-.453	1442	.320	-.530

GRAND WEIGHTED AVERAGE N=26,290 A=.331 B=-.463

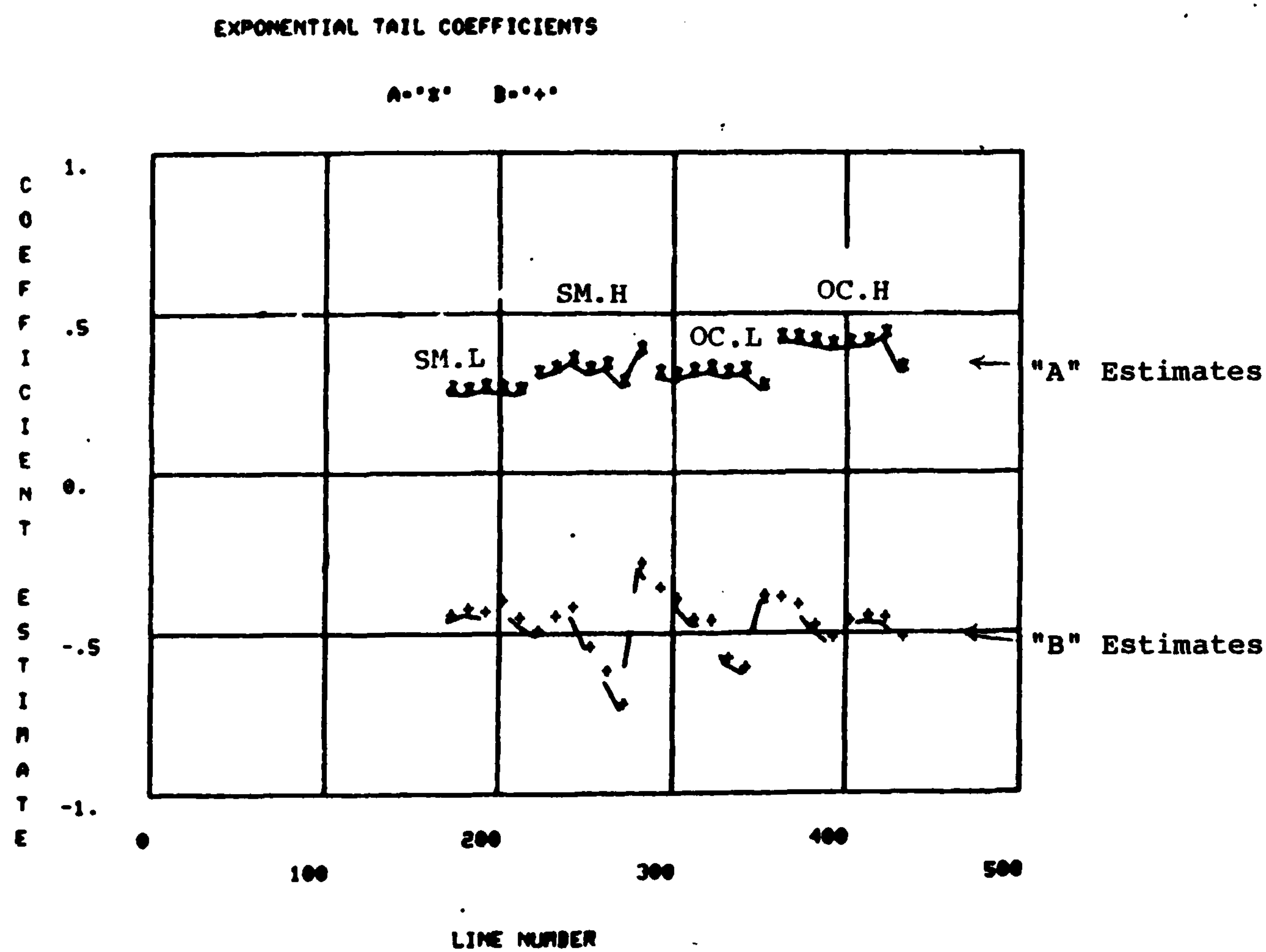

Figure III-8. Coefficient Estimates for the Model P[Z≥z] = A exp(Bz).

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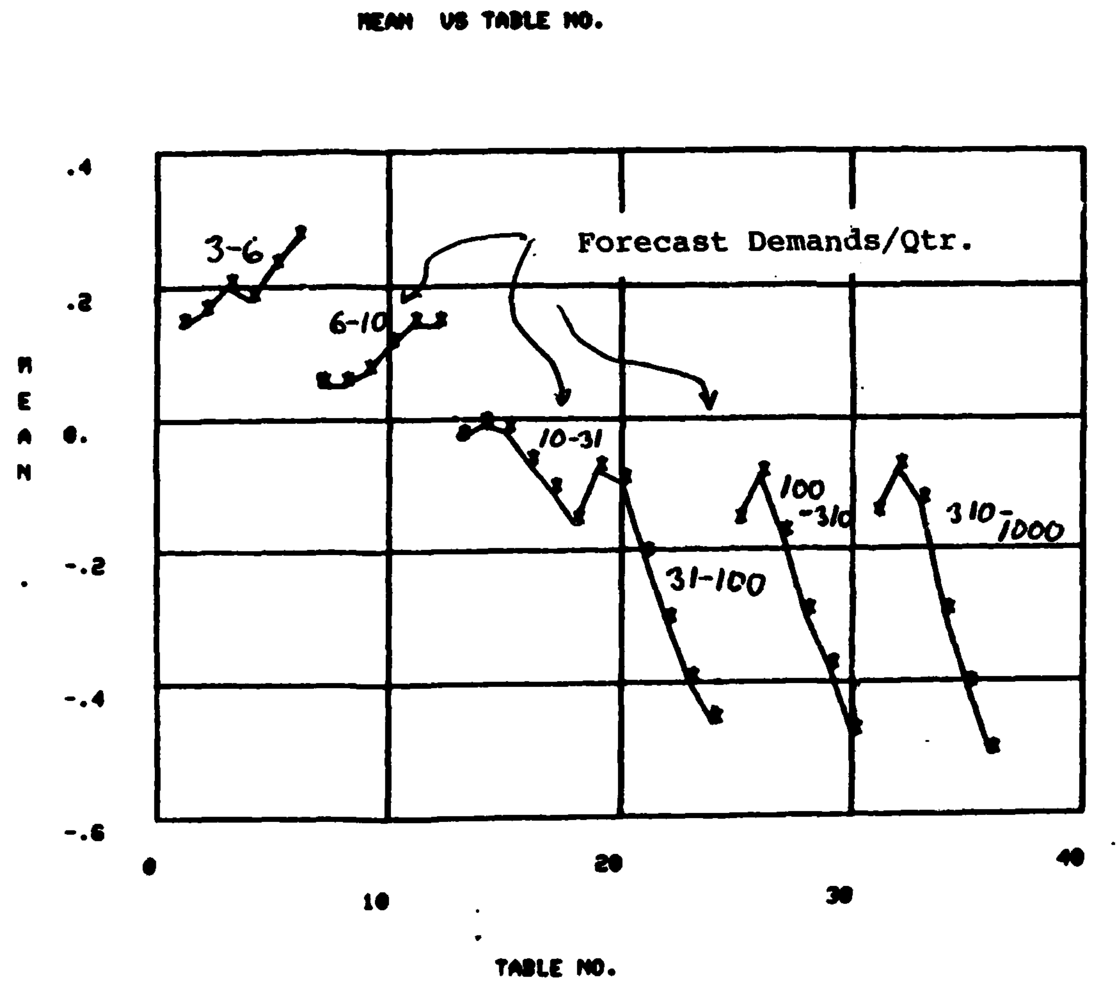

Figure III-9. Mean Values for CZi/k
for i=1,2,...6
for Sample OC.H

CZ/K associated with forecasts which are at the three to six units per quarter forecast range. The six points represent lead times of one, two,..., six quarters, respectively. Note that for these points, the mean CZ/K value is positive--that is, actual demands tend to be greater than the forecasted value. Further, this mean error increases as the lead time increases. The second group of six points corresponds to forecasts in which the forecast demand rate is in the range of six to ten units per quarter. Note that a similiar phenomena occurs here; however, the mean CZ/K values are less. On the other hand, observe that for the high demand rate catagories, the mean CZ/K values are negative --indicating actual demands are on the average less than the values forecast. Also note that this over-forecasting charateristic becomes more severe as the forecast horizon lengthens.

Figure III-10 plots the standard deviation of the normalized value CZ/K verses the Histogram number. Notice that the standard deviation increases as the lead time increases, but that the standard deviation is similiar across all demand rate catagories. If demand were in fact independently distributed, the standard deviation of CZ/K should not change as a function of lead time. Finally, Figure III-11 presents a plot of the squared value of skewness verses the kurtosis associated with each of the forcast catagories. As shown in the figure, items with demand rate forecasts in the 3-6 unit range have skewness/kurtosis characteristics which lie between those associated with gamma and log-normal

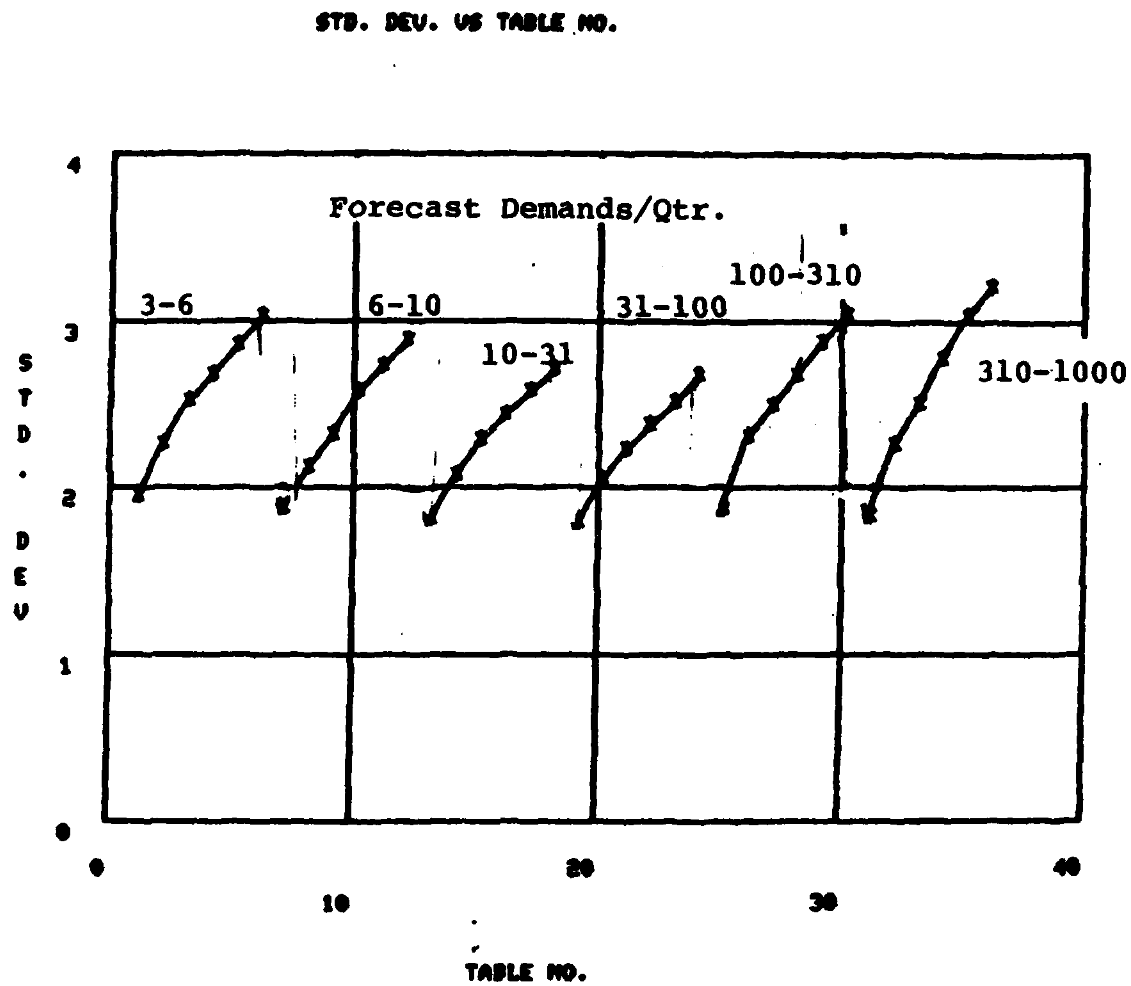

Figure II-10. Standard Deviation of CZi/k for Leadtimes i=1,2,...6 for Sample OC.H

Skewness² vs Kurtosis

Forecast	No. of Observations
3-6	1184
6-10	1172
10-31	3000
31-100	2619
100-310	1274
310-1000	565

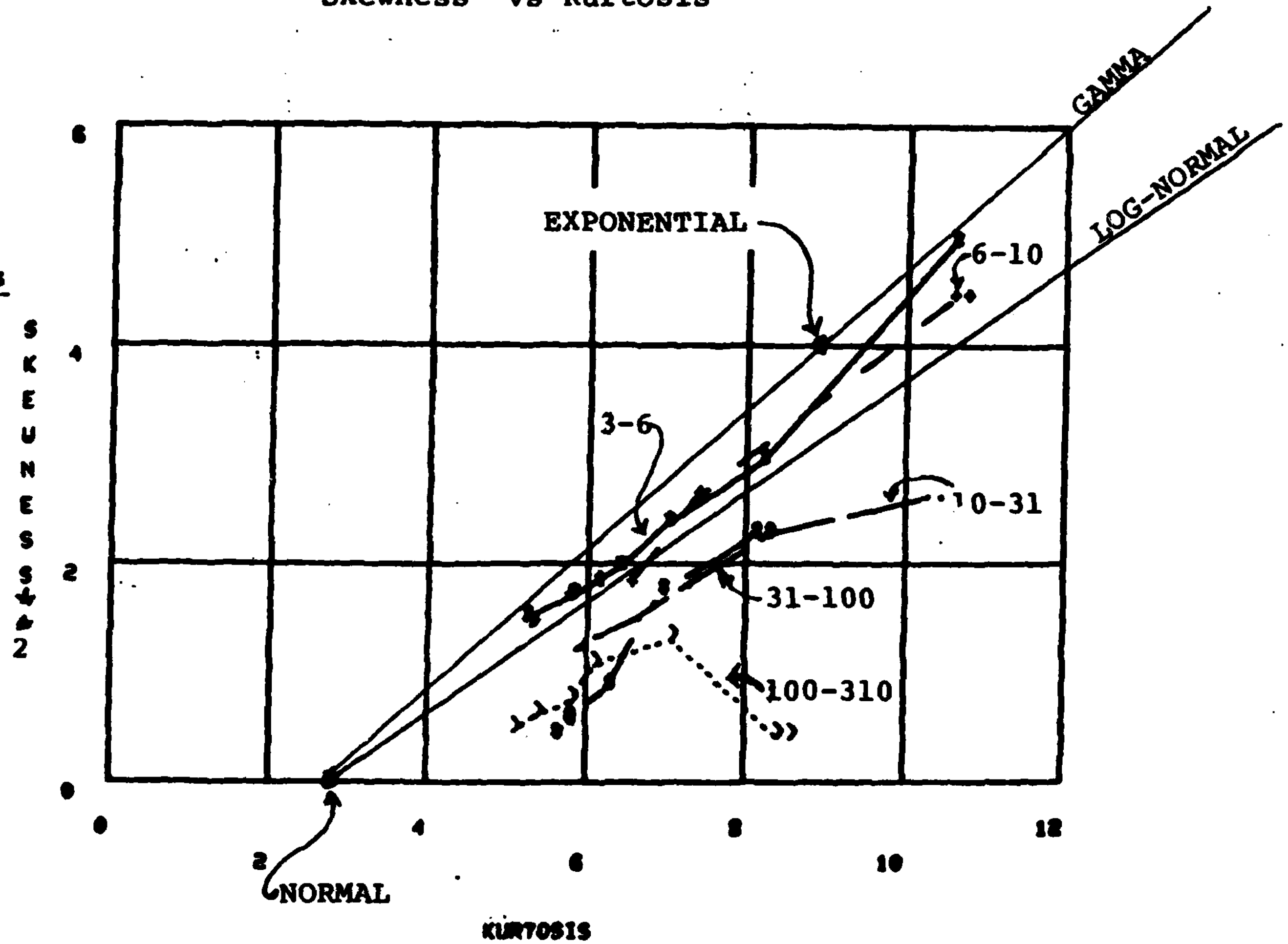

Figure III-11. Skewness² vs Kurtosis for OC.

functions. This suggests that perhaps a gamma or log-normal distribution may be a useful approximation to the actual CZ/K distribution for items in this range. Note that as the demand rate catagory increases, the resulting distributions tend to be less skewed and the kurtosis measures also decrease slightly. However, the skewness/kurtosis values still depart significantly from the values that would be expected from a normal distribution of forecast errors.

Section iV

Analysis of the Distribution of Demand per Quarter for Low Activity Items

As noted in earlier sections, we observed significant differences in forecast error distributions among items grouped on the basis of demand activity in a base period. Analysis of item demand histories shows that many low demand D062 items have many periods of zero demands but, when demands do occur, they are often of large magnitude relative to the average usage rate. On the other hand, high activity D062 items appear to be less erratic though still highly variable. Consequently, it appears that an optimum D062 inventory management system might utilize different methods in modeling high versus low activity items. For example, in many commercial inventory systems, the Poission distribution is used to model low activity item demands, while the Normal distribution is used to model high activity demand processes. In Section III, we developed an empirical model of the probability distribution of forecast errors for high activity items. In this section, we report the results of similar efforts to develop an appropriate model for the distribution of forecast errors for low activity items.

Initial Studies

We began our studies in this area by asking several questions. In particular, we asked "Are there significant differences in the distribution in actual demands in a given quarter:

a. As the lead time increases?

b. As the number of demands observed in the last eight quarters increases?

c. As the forecast interval changes ?"

To answer these questions, we first developed histograms of the net demands observed in a given quarter given that a total of T demands have been observed in the past eight quarters. For example, suppose that a total of T = 10 demands were observed during the past eight quarters for a given item. We then tabulated that number of demands which occurred one

quarter in the future for this item. A similar tabulation was done for all items in which a total of T = 10 demands had been observed in the last eight quarters. This provided a histogram of actual net demand one quarter in the future given a total of 10 demands had been observed in the past eight quarters. This histogram construction process was then repeated for forecasts of demand two quarters in the future, three quarters in the future, etc. Let X_i denote the number of demands actually observed in period i. Then the above tabulation process provides an empirical measurement of $P(X_i/T)$, the conditional probability of observing X_i units in quarter i given a total of T units were demanded in the past eight quarters.

To obtain answers to the above questions, we first developed histograms of $P(X_i/T)$ for all items with total CY71-72 demands T of 1, 2, 4, and 8 units, respectively. For each group of items, we developed separate histograms for forecast leads times of i = 1, 4, and eight quarters, respectively. This resulted in a total of (4 base demands) x (3 lead times) = 12 separate histograms for a given item sample. This histogram building process was then repeated using the intervals CY73-74, CY75-76, and CY77-78 as the base period. This resulted in a total of 48 separate $P(X_i/T)$ histograms for each item sample.

We next carefully analyzed and compared these histograms. We found that within a given item sample, demand per quarter tends to be identically distributed for a given base demand T. That is, the observed cumulative distribution functions for $P(X_i/T)$ are very similar for all lead time values i (i= 1,4,8) and for all the base period intervals. However, items with very few base period demands T have significantly different $P(X_i/T)$ curves than items with high levels of past demand activity.

Based on the above results, we developed another set of histograms. In this effort, we assumed that $P(X_i/T)$ was identically distributed across lead times i and across forecast intervals. Histogram 1 was used to tabulate actual quarterly net demand for items with total (past) base period demands of T=1 unit, while histogram two tabulated net quarterly demands for items with past demands T=2 units. A total of 48 histograms,

corresponding to total base period demands of T=1, 2, ..., 48 units, respectively, was constructed for each item sample.

We next sought analytical approximations to these empirical results.

The Poisson Model

The Poisson probability distribution is often used to model the distribution of demands for low activity items. Let R denote the average quarterly demand rate for a given item. Then if demands are Poisson distributed, the probability of x units being demanded in a given quarter is given by

$$p(x) = \frac{R^x \exp(-R)}{x!} \quad x=0,1,2,\ldots$$

If T units have been demanded in the past eight quarters, an estimate of R is T/8.

Figures IV-1 and IV-2 plot the probability of zero demands in a given quarter for samples SM.L and OC.L for both the Poisson distribution and for the empirical $P(X_i|T)$ histograms. As shown in the figures, observed D062 demands have a significantly higher probability of zero demands than is predicted by the Poisson model. For the Poisson model, as the demand rate increases, P(X=0) becomes very small. However, for the observed D062 demands, P(X=0) was 30% or higher even when 40-50 units had been demanded in the past two years.

As a result of these observations, we concluded that the Poisson distribution is not a good model for describing the demand process for low activity D062, items.

Exponential and Split Poisson Approximations

Since the observed data was clearly not Poisson distributed, we sought convenient mathematical approximations to the observed data. Two approximation methods were tested. First, we obtained an exponential approximation by fitting an exponential curve through the P(X=0) and P(X=9) points on the observed cumulative distribution function for net demand. As a second approximation, we fit a probability model that consisted of a spike of probability at X=0 (corresponding to zero demands), with the remaining

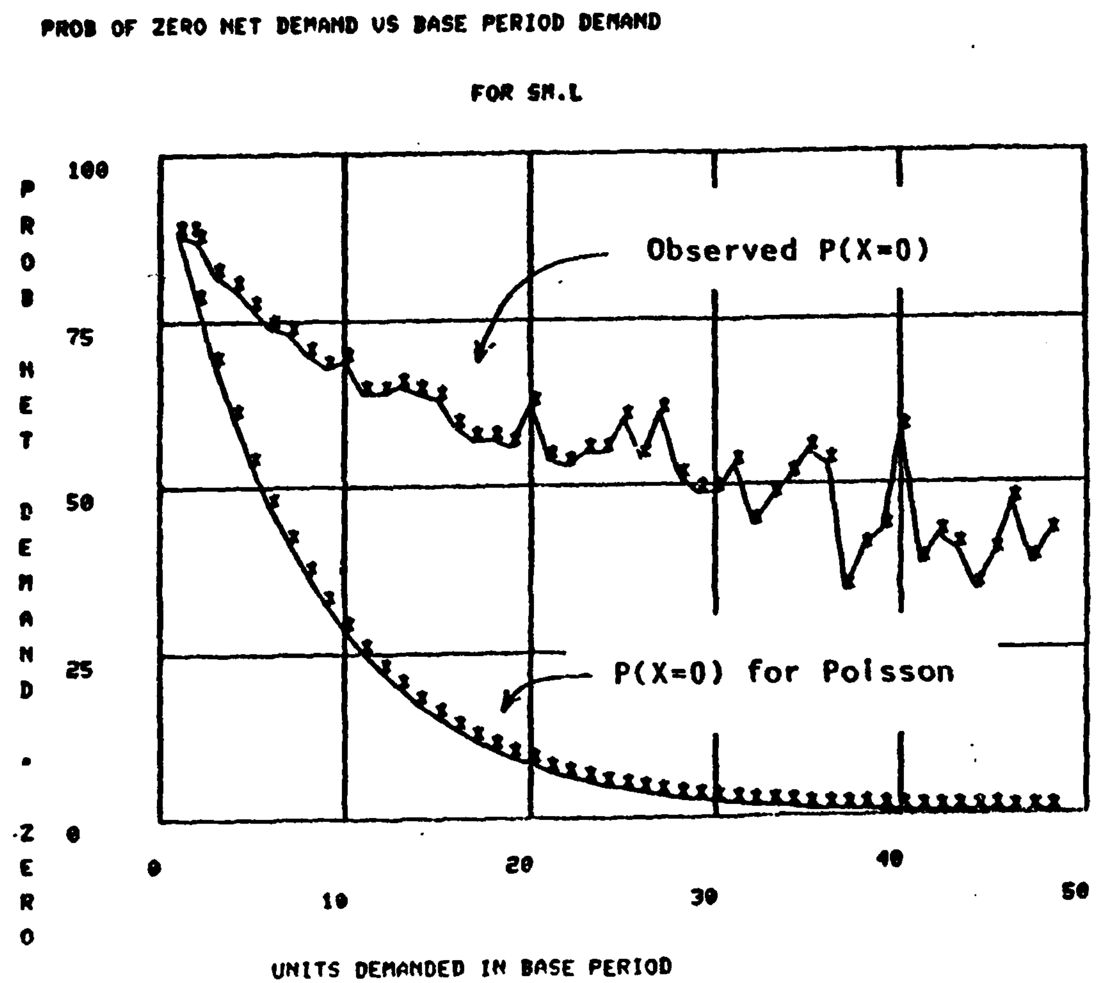

Figure IV-1. Comparison of Observed and Poisson Probabilities for P(X=0) for Sample SM.L.

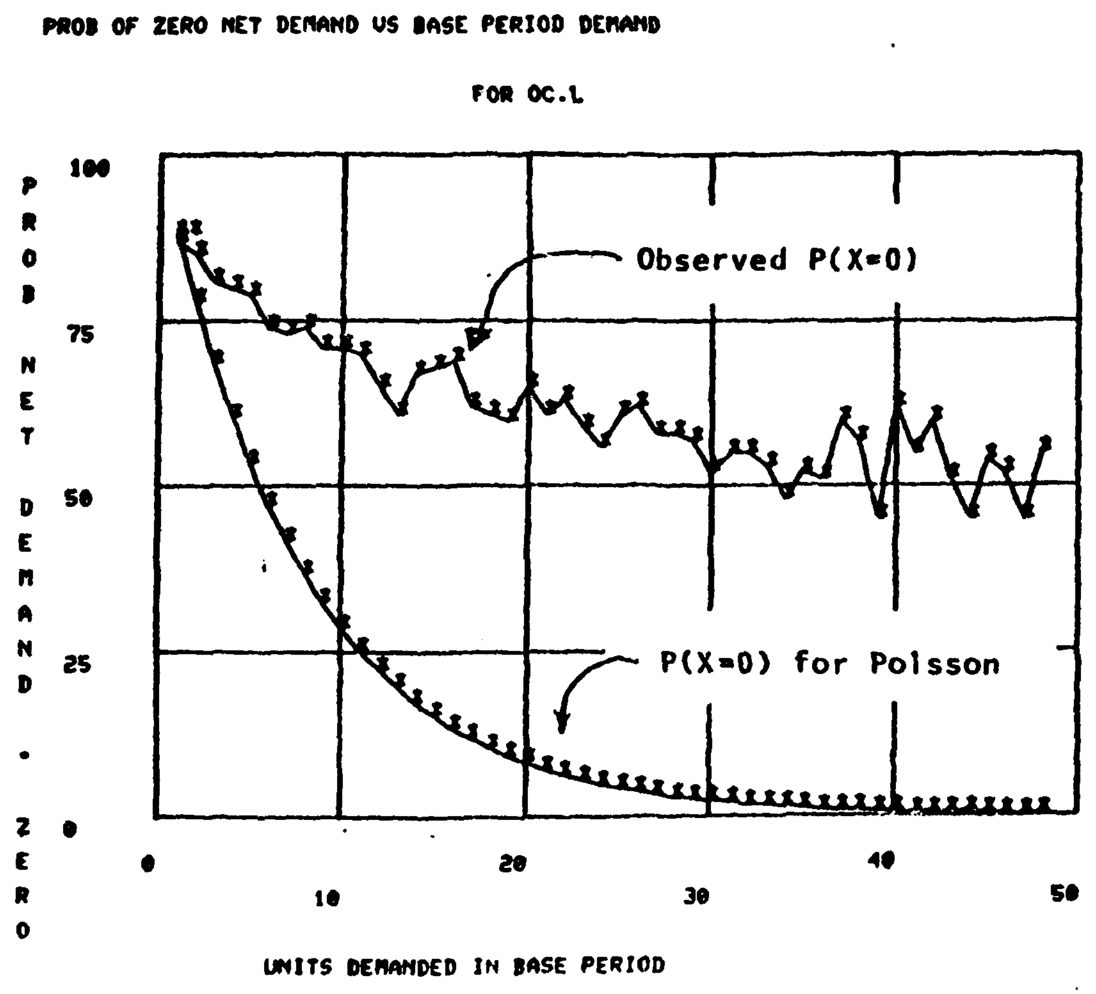

Figure IV-2. Comparison of Observed and Poisson Probabilities for P(X=0) for Sample OC.L.

probability allocated according to the Poisson formula. We refer to this second approximation as the "split Poisson," since we have split the probability values between the $P(X=0)$ point and the Poisson model.

Testing the Approximation

To test the above approximations, we developed a series of graphs comparing the observed cumulative distribution functions of net demand and the two approximations described above. Because of the limited amount of data available, the CDF for any particular $P(X \mid T)$ curve was often rather unstable. To avoid this problem, we constructed an average curve by pooling the data for several histograms. We then constructed approximations to the average curve.

Our results for histograms 1 thru 6 are presented in Figure IV-3. To the left of this figure, we present the observed cumulative distribution functions for the histogram in which the demands observed during the last eight quarters equal to one unit, two units, . . ., six units, respectively. We then computed the average of these six curves, and plotted the average curve as well as the exponential and spilt Poisson approximations to this curve. These latter results are shown on the right hand side of Figure IV-3. As shown in the figure, the exponential curve provides a reasonable approximation to the average curve, while the spilt Poisson provides a less desirable fit. The coefficients associated with the estimation equations are shown at the bottom of this figure. The coefficients a and b correspond to the exponential approximation model $P(X \leq x) = 1 - a \exp(b x)$, while the parameter c is a parameter of the spilt Poisson model.

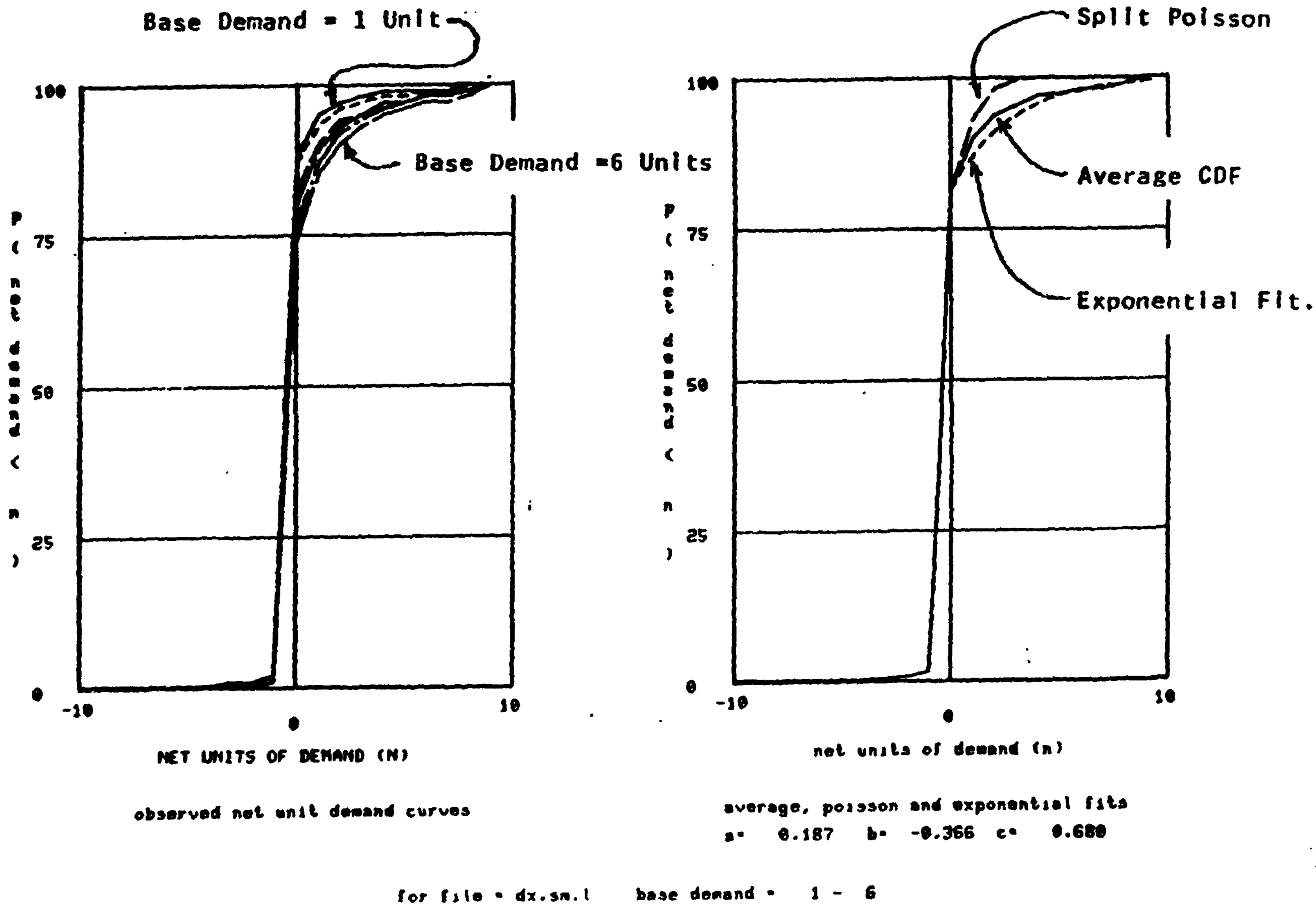

Figure IV-3. Observed Cumulative Distribution of Net Demand for Sample SM.L and Base Demand of 1-6 Units.

Figures IV-4 thru IV-10 present similar results for histograms 7 thru 48. In each case, on the left hand side we display the observed cumulative distribution functions for net demand given that demand in the past eight quarters equals to a specified value T, while on the right hand side we compare the average of these six curves with the exponential and spilt Poisson approximations. Notice that as the number of demands in the base period increases, the exponential approximation becomes a better fit to the actual CDF. On the other hand, the fit of the split Poisson becomes very poor as the base period demands increase. Consequently, we conclude that the exponential approximation would be a reasonable model for demand in a given quarter for low demand D062 items and that the spilt Poisson is not useful for this purpose.

Table IV-1 presents the exponential coefficient estimates associated with the Oklahoma City and Sacramento ALC low activity samples while Figure IV-11 presents a graph of this data. As shown in the figure, the coefficient estimates increase as the number of units in the past eight quarters increase. Least squares-fit equations which might be used to obtain A and B coefficients for a given value of base demand T are presented at the bottom of Table IV-1.

AD-A124 617 A STATISTICAL ANALYSIS OF THE DISTRIBUTION OF D062 DEMAND IN A GIVEN LEADTIME(U) DECISION SYSTEMS BEAVERCREEK OH W S DEMMY SEP 81 WP-81-03 2/2

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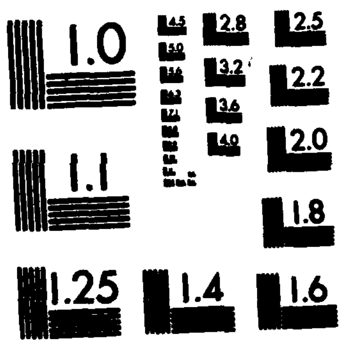


MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

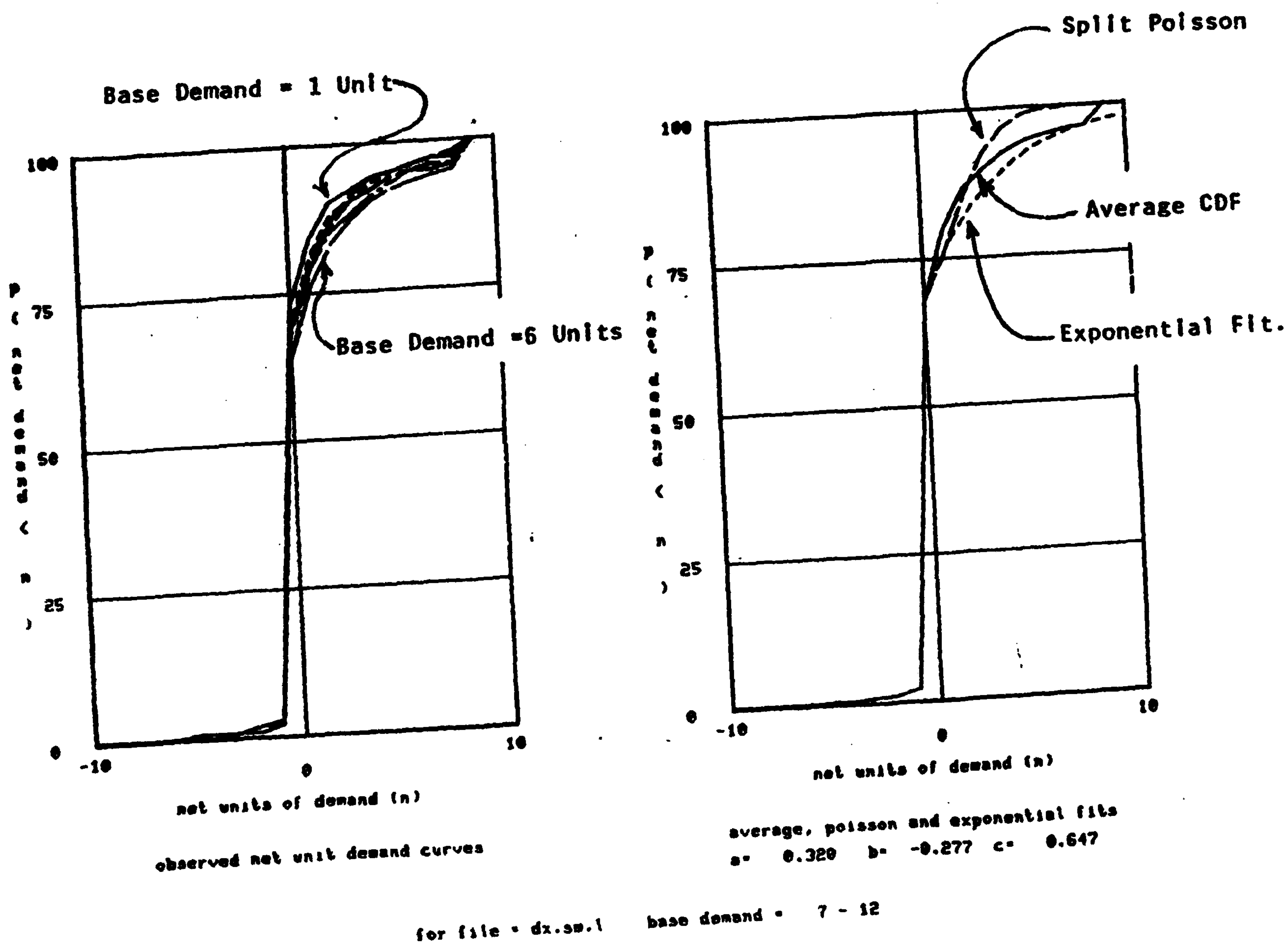
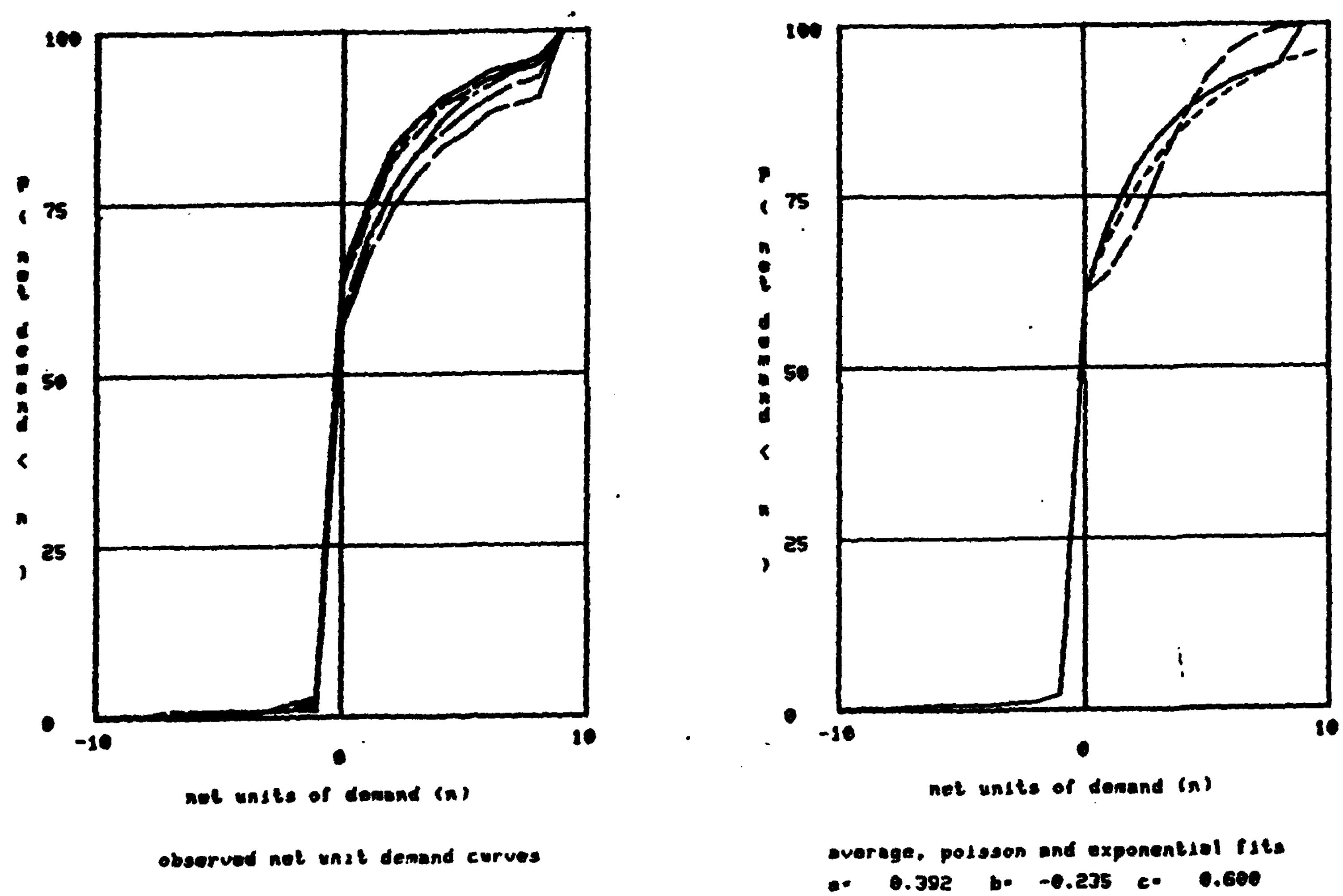


Figure IV-4. Observed Cumulative Distribution of Net Demand for Sample SM.L and Base Demand of 7-12 Units.



for file = dx.sm.l base demand = 13 - 18

Figure IV-5. Observed Cumulative Distribution of Net Demand for Sample SM.L and Base Demand of 13-18 Units.

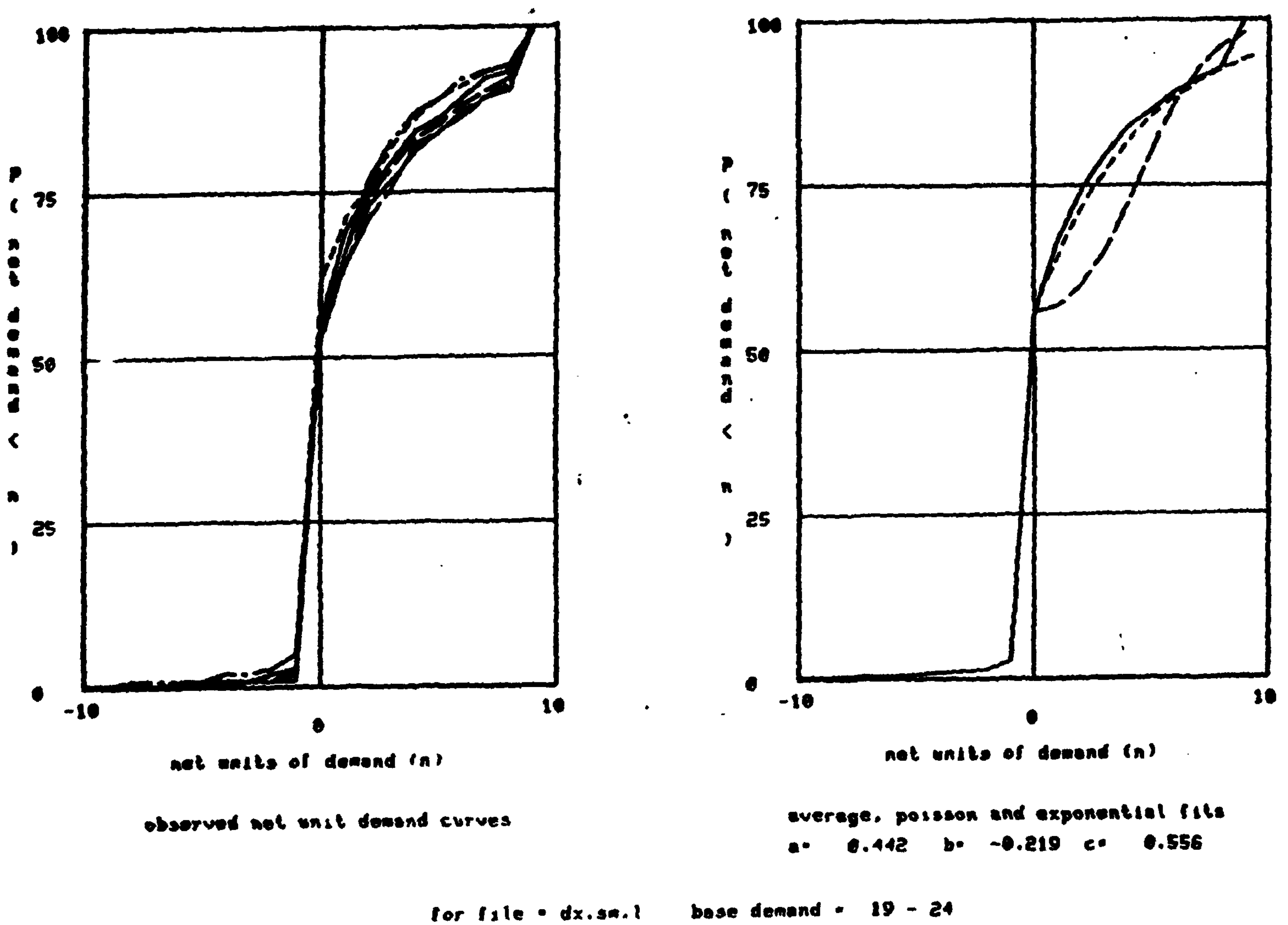
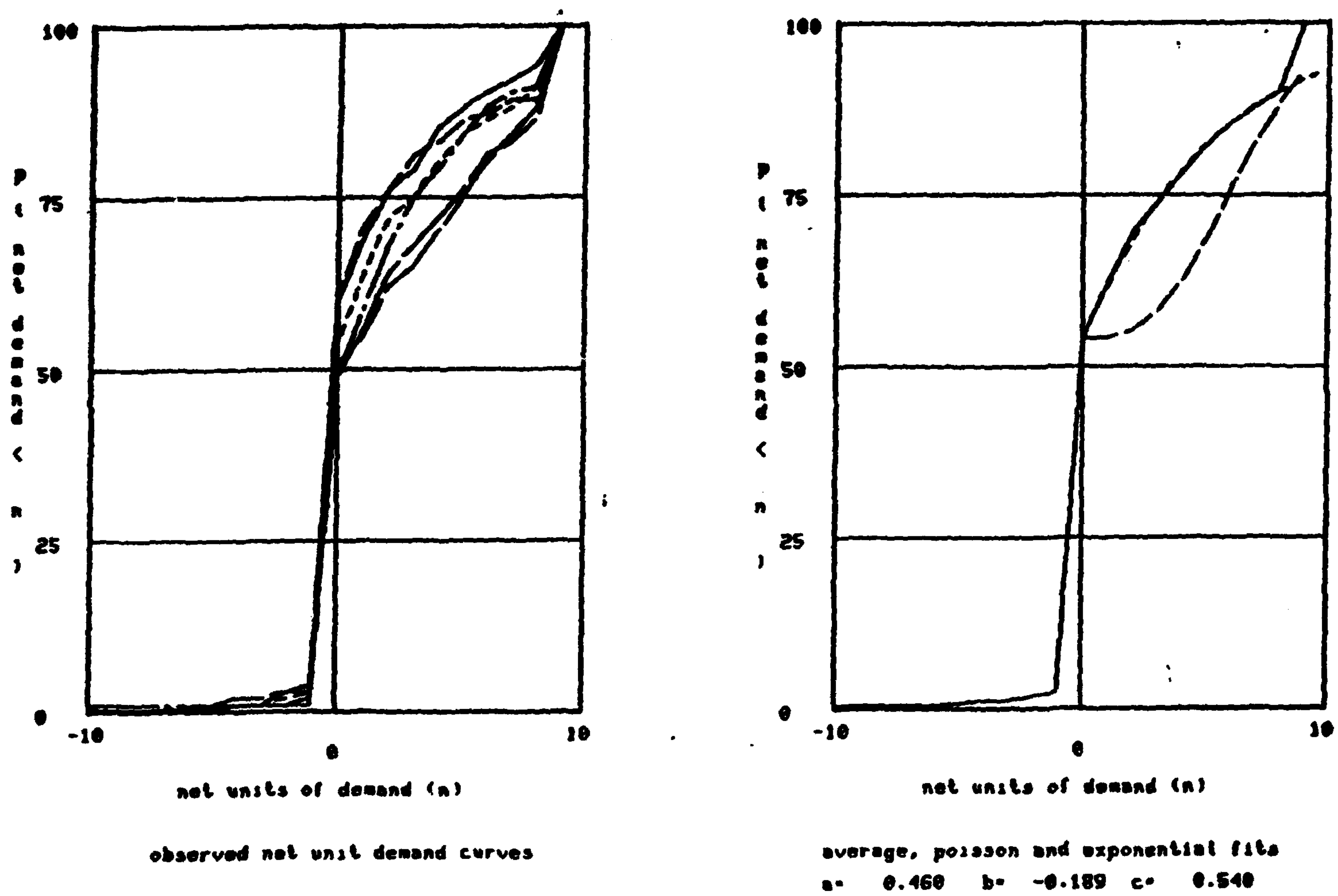
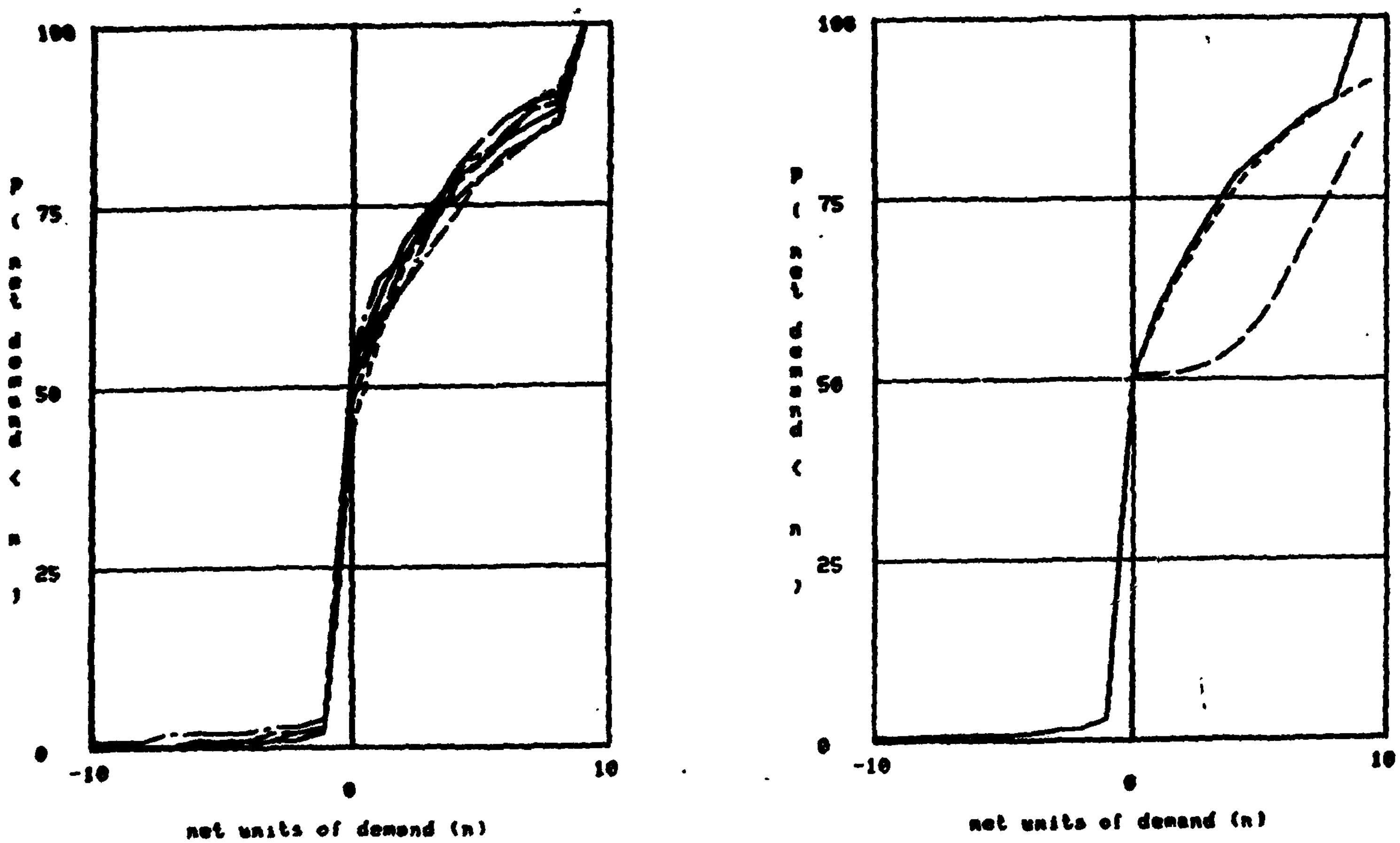


Figure IV-6. Observed Cumulative Distribution of Net Demand for Sample SM.L and Base Demand of 19-24 Units.



for file = dx.sm.l base demand = 25 - 30

Figure IV-7. Observed Cumulative Distribution of Net Demand for Sample SM.L and Base Demand of 25-30 Units.



observed net unit demand curves

average, poisson and exponential fits
a= 0.493 b= -0.182 c= 0.507

for file = dx.sm.l   base demand = 31 - 36

Figure IV-8. Observed Cumulative Distribution of Net Demand for Sample SM.L and Base Demand of 31-36 Units.

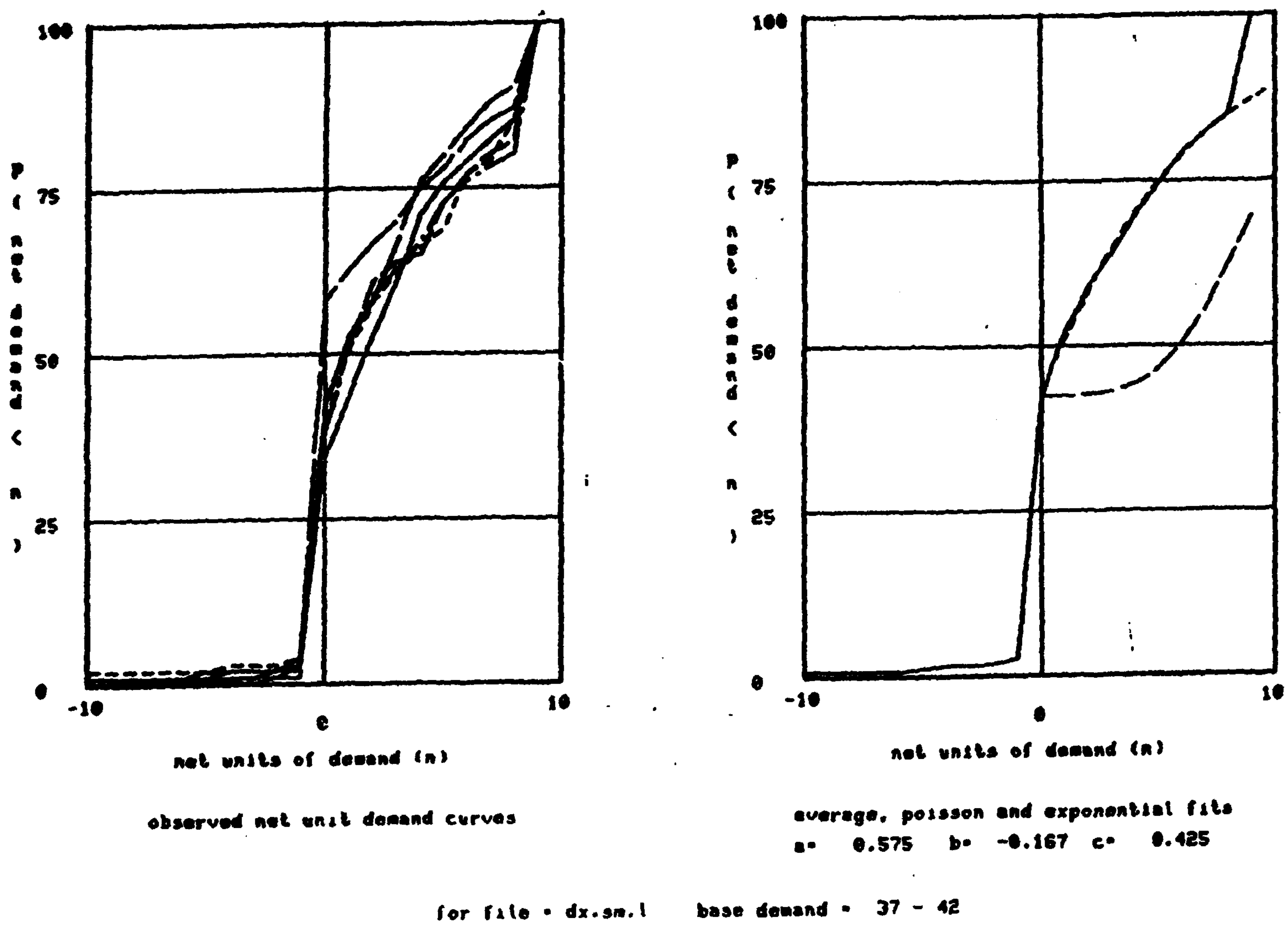


Figure IV-9. Observed Cumulative Distribution of Net Demand for Sample SM.L and Base Demand of 37-42 Units.

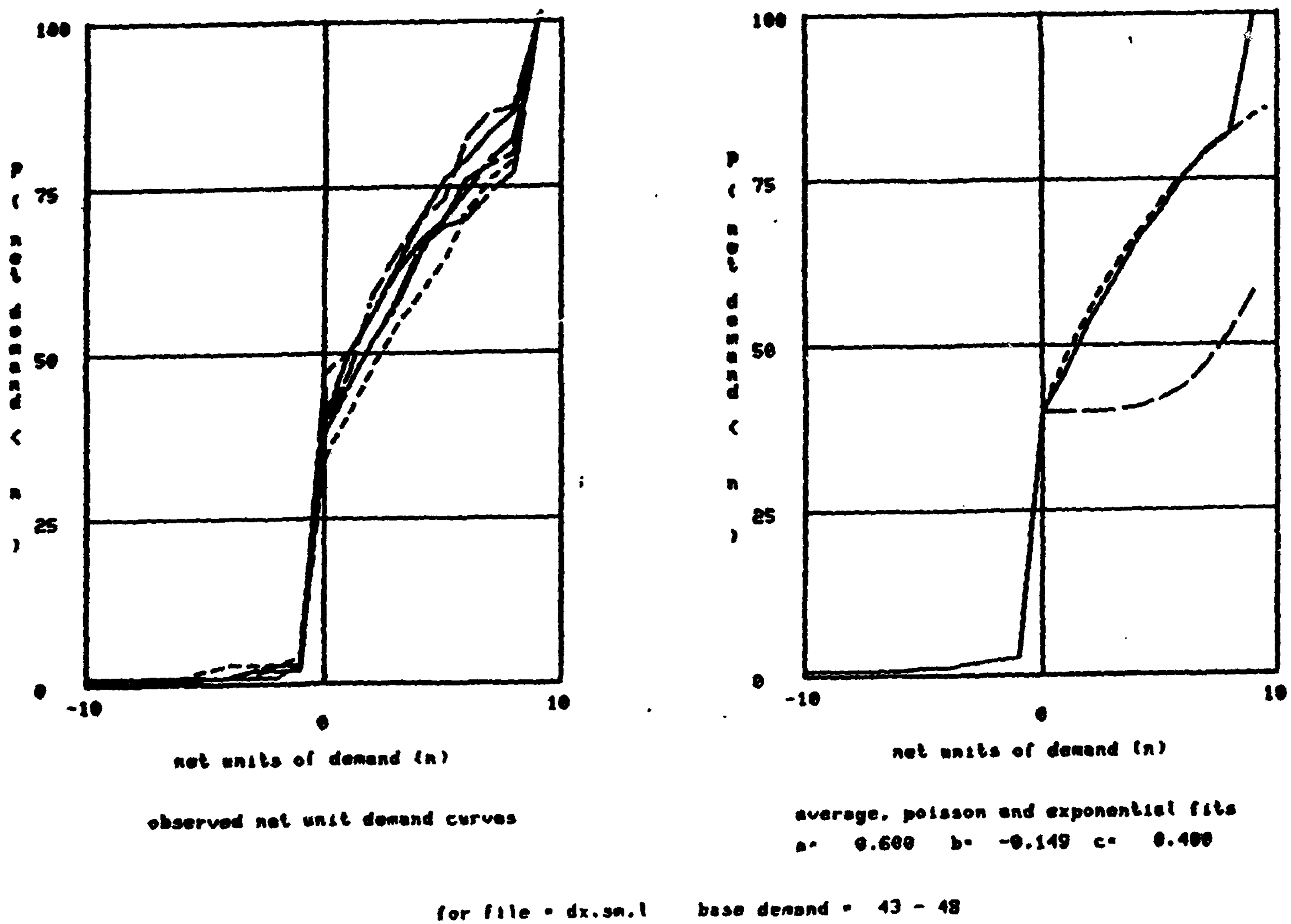


Figure IV-10. Observed Cumulative Distribution of Net Demand for Sample SM.L and Base Demand of 43-48 Units.

Table IV-1

Exponential Coefficient Estimates
for
Low Activity Items

Model: $P(\text{Net Demand} \leq x) = 1. - A*\exp(B*x)$

Total Units Demanded in Base Period	Midpoint	OC.L A	OC.L B	SM.L A	SM.L B
1- 6	3.5	.19	-.281	.187	-.366
7-12	9.5	.293	-.239	.320	-.277
13-18	15.5	.353	-.212	.392	-.235
19-24	21.5	.393	-.184	.442	-.219
25-30	27.5	.420	-.164	.460	-.189
31-36	33.5	.477	-.164	.493	-.182
37-42	39.5	.435	-.158	.575	-.167
42-48	45.5	.497	-.151	.600	-.149

Least Squares Estimate:

$A = .2239 + 0.0065\ T$ $A = .2155 + 0.0089\ T$

$B = -.266 + 0.0029\ T$ $B = -.3328 + 0.0045\ T$

where T = Midpoint of Base Demand

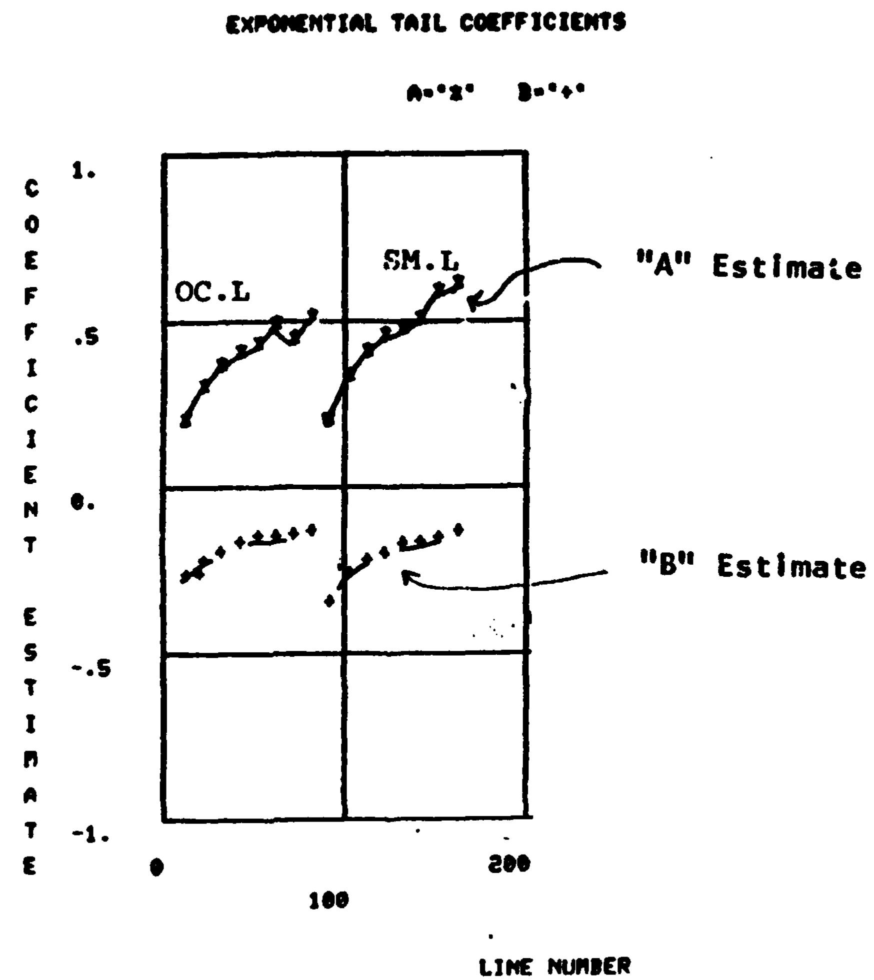


Figure IV-11. Plot of OC.L and SM.L Exponential Fit Coefficient Estimates

7/11/81

SECTION V

Summary of Results

This paper has presented statistical data describing the distribution of forecast errors associated with current D062 demand forecasting procedures. Section I provides a detailed review of these procedures while Section II describes the results of preliminary studies of the nature of BOQ demand forecast errors. Finally, Sections III and IV present refined models for the distribution of forecast errors for high and low activity items, respectively.

In Section III, we studied forecast errors for items with forecast demand rates of three units per quarter or more, while Section V describes the error distributions for lower activity items. In Section III, we found that for positive standardized forecast errors (i.e., errors in which demand exceeds the forecast value), an exponential model provides an excellent fit to observed error statistics. An exponential approximation may also be useful in approximating the distribution for negative forecast errors, but the fit is not as good for this side of the cumulative error distribution curve. Note that if leadtimes are considered fixed and safety levels are restricted to be non-negative, then the positive error portion of the error distribution function is all that is needed to set cost-effective safety levels. For this situation, we have an excellent fit.

In Section IV, we also found an useful exponential approximation. We defined $P(X_i|T)$ as the conditional distribution of net demand in a given quarter i given that a total of T units of demand were observed in the p;ast eight quarters. We found that $P(X_i|T)$ appears to be identically distributed across leadtime periods i and across forecast intervals. Further, we found the model $P(X \leq x|T) = 1 - a \exp(bx)$ for $X = 0, 1, 2, \ldots$ to be a good approximation to the observed $P(X_i|T)$ distributions. Analytical or numeric calculation of convolutions of this model might then be used to estimate the distribution of cumulative net demand for any given leadtime.

References

1. Demmy, W. Steven. Statistical Characteristics of Forecasting Techniques for D062 Economic Order Quantity Items. TR-79-02, Decision Systems, 2125 Crystal Marie Dr., Beavercreek, Ohio 45431, May 1979, 128 pp.

2. Demmy, W. Steven. A Comparison of Forecasted and Actual Flying Programs for CY 1973-1979. Working Paper WP-80-06, Decision Systems, 2125 Crystal Marie Drive, Beavercreek, Ohio 45431, October 1980, 44 pp.

3. Demmy W. Steven. Plots of CY 71-79 Demands and Returns for a Sample of Sacramento ALC D062 Items. Working Paper WP-81-01, Decision Systems, 2125 Crystal Marie Drive, Beavercreek, Ohio 45431, May 1981, 119 pp.

4. Hayya, Jack C. Leadtime Variability in Inventory Requirements Projections, Phase II , Technical Report and Summary. AF Contract #33615-79-C-5143; Item #004, 1962 Norwood Lane, State College, Pennsylvania 16801, June 30, 1980, 71 pp.

Appendix A

Analysis of Variance

of

Period Errors Zi

Table A-1.1. Breakdown by Base Period for All Items in Sample OC.H.

Analysis of Variance for Difference Between Forecast Base Periods
For Sample OC.H.

SNUMB A3797 DATE 4-4-81 SBI	PERIOD NO		Z_1	Z_2	Z_3	Z_4	Z_6	Z_8	Z_{10}	Z_{12}		NO. OF FORECASTS
BASE PERIOD												
	1		.05	-.00	-.07	-.17	-.36	-.46	-.26	-.28		3154
	2		.19	.08	.11	.10	.10	.15	.11	.08		3154
	3		-.08	.04	-.11	-.01	.11	-.10	-.06	-.35		3154
	4		-.00	.15	.05	-.10	.04	0.	0.	0.		3154
OVERALL MEAN			.0390	.0994	-.0027	-.0482	-.0255	-.1060	-.0526	-.1365		12616
" STD DEV			1.8804	2.0208	2.0194	2.0366	2.1283	1.7711	1.8926	1.8333		
ANOVA RESULTS												
BETWEEN GROUPS F			13.107	4.713	8.726	11.268	36.045	69.970	22.515	42.371		
" " SIGNIFICANCE			0.0000	.0028	0.	0.	0.	0.	0.	0.		
r^2			.0008	.0008	.0001	0.	.0042	.0051	.0012	.0006		
ETA^2			.0031	.0011	.0021	.0027	.0086	.0163	.0053	.0100		

Table A-1.2. Breakdown by Base Period for All Items in Sample OC.L.

Analysis of Variance for Difference Between Forecast Base Periods For Sample OC.L.

SNUMB HD03I DATE 4-9-81 S.B1	PERIOD No		Z_1	Z_2	Z_3	Z_4	Z_6	Z_8	Z_{10}	Z_{12}		No. OF FORECASTS
BASE PERIOD												
	1		.09	.05	.05	-.06	-.09	-.07	-.05	-.10		4828
	2		.10	.07	-.00	-.02	.03	-.01	.01	.00		4749
	3		-.19	-.12	-.14	-.11	-.04	-.17	-.09	-.16		4756
	4		-.01	.10	.02	.08	.05	0.	0.	0.		4782
OVERALL MEAN			0.0002	0.0272	-0.0184	-0.0529	-0.0115	-0.0692	-0.0345	-0.0682		19115
" STD DEV			1.6452	1.7114	1.6834	1.6268	1.6893	1.4601	1.5053	1.4594		
ANOVA RESULTS												
BETWEEN GROUPS F			33.735	17.281	13.308	4.631	8.130	13.376	5.715	15.330		
" " SIGNIFICANCE			0.	0.	0.	0.0031	0.	0	0.0007	0.		
Y^2			0.0018	0.	0.0002	0.0001	0.0006	0.	0	0.0002		
ETA^2			0.0053	0.0027	0.0081	0.0007	0.0013	0.0021	0.0009	0.0084		

Table A-1.3. Breakdown by Base Period for All Items in Sample SM.H.

Analysis of Variance for Difference Between Forecast Base Periods
For Sample SM.H.

SNUMB A3687 DATE 4-4-81 S.B1	PERIOD No		Z_1	Z_2	Z_3	Z_4	Z_6	Z_8	Z_{10}	Z_{12}		No. of Forecasts
BASE PERIOD												
	1		-.18	-.37	-.28	-.42	-.54	-.37	-.50	-.36		630
	2		-.24	-.03	.06	-.12	-.19	-.02	-.29	-.10		630
	3		-.43	-.75	-.58	-.56	-.56	-.60	-.69	-.87		630
	4		.06	.06	-.09	-.14	-.27	0.	0.	0.		630
OVERALL MEAN			-.0789	-.2757	-.2253	-.2534	-.3950	-.2532	-.3729	-.3372		2520
" STD DEV			1.9314	1.9776	1.9149	1.9961	1.8272	1.7016	1.6980	1.8112		
ANOVA RESULTS												
BETWEEN GROUPS F			14.806	22.880	13.355	15.166	6.653	18.631	19.352	29.165		
" " SIGNIFICANCE			0.	0.	0.	0.	.0002	0.	0.	0.		
r^2			0.	.0011	0.	.0001	.0008	.0013	.0052	.0004		
ETA^2			.0173	.0258	.0157	.0178	.0079	.0217	.0226	.0836		

Table A-1.4. Breakdown by Base Period for All Items in Sample SM.L.

Analysis of Variance for Difference Between Forecast Base Periods For Sample SM.L.

SNUMB A376T DATE 4-4-81 S.B1	PERIOD No		Z_1	Z_2	Z_3	Z_4	Z_6	Z_8	Z_{10}	Z_{12}		No. of FORECASTS
BASE PERIOD												
	1		.05	-.11	-.04	-.02	-.08					6601
	2		.16	.02	.05	.04	-.06					6601
	3		-.22	-.28	-.24	-.27	-.24					6601
	4		.02	.09	.09	.02	.00					6601
OVERALL MEAN			.0069	-.0713	-.0344	-.0563	-.0981					26404
" STD DEV			1.7762	1.7015	1.7747	1.7929	1.7080					
ANOVA RESULTS												
BETWEEN GROUPS F			55.640	62.247	48.754	43.968	24.310					
" " SIGNIFICANCE			0.	.9750	0.	0.	0.					
Y^2			.0008	.0004	.0001	.0001	0.					
ETA^2			.0063	.0070	.0055	.0050	0.0028					

Table A-2.1. Breakdown by Aircraft Code for All Items in Sample OC.H.

Analysis of Variance for Differences Between Aircraft Groups
For Sample OC.H.

SNUMB A379T
DATE 4-4-81
S. B1

CODE	WEAPON	WEAPON No.		Z_1	Z_2	Z_3	Z_4	Z_6	Z_5	Z_{11}	Z_{12}		No. of Forcasts
9999	COMMON	C		.04	.07	-.02	-.01	.03	-.05	.01	-.03		2004
1012	B52	1		.22	.02	.05	-.08	.07	-.10	-.25	-.22		2284
3072	B57	2		.07	.02	-.00	.21	-.22	-.48	-.09	.02		116
1062	C118	3		.65	.07	.02	.03	.22	-.00	.00	-.16		124
9652	C121	4		.61	-.26	.00	-.19	-.03	-.14	-.48	-.48		44
7132	C123	5		.16	.04	-.30	-.57	-.26	-.38	-.41	-.45		120
4002	C130	6		-.34	-.09	-.12	-.29	-.16	-.23	-.18	-.33		1076
1192	C135	7		-.10	.05	-.01	-.21	-.12	-.15	-.08	-.29		740
4762	C141	8		.14	.07	.02	.11	-.04	.14	.07	.09		720
3922	F4	9		.16	.37	.20	.16	.23	.12	.28	.15		1968
7202	F5	10		-1.24	-.61	-1.13	-.50	-1.37	-.56	-.80	-.26		36
3052	F100	11		-.65	.01	-.08	-.07	-0.11	-.18	-.38	-.50		328
2172	F101	12		-.18	.05	-.13	-.04	-0.07	-.22	-.25	-.27		260
2012	F102	13		.02	-.10	-.08	.26	-0.01	-.25	-.44	-.30		180
2032	F104	14		-.82	-.84	-.60	-1.00	-0.78	-1.14	-.78	-1.00		240
3062	F105	15		-.12	-.09	-.05	-.30	-0.34	-.09	-.05	-.02		364
2016	F106	16		.03	-.01	.08	.10	-.03	-.33	-.06	-.27		660
9682	T38	17		.24	.26	-.18	.09	-.20	.06	.11	-.07		216
9242	T37	18		.40	.24	-.11	.48	-.09	-.12	.20	.27		96
7202	T38	19		.13	-.01	-.03	-.28	-.43	-.16	-.18	-.14		288
4522	T39	20		.09	-.09	-.13	-.21	-.28	.17	-.37	-.29		180
3242	TF111	21		.04	.52	-.05	-.02	-.07	-.11	.59	.13		480
9602	CT29	22		-.00	.01	-.13	-.06	.06	-.27	-.36	-.51		172
OVERALL MEAN				.0390	.0794	-.0027	-.0482	-.0265	-.1060	-.0626	-.1355		12616
STD. DEV.				1.8716	2.0121	2.0175	2.0292	2.1292	1.7950	1.8811	1.8298		
ANOVA RESULTS													
BETWEEN GROUPS F				8.11	6.53	3.124	6.069	5.252	7.668	10.946	8.855		
" " SIGNIFICANCE				0.	0.	0.	0.	0.	0.	0.	0.		
r^2				.0067	.0002	.0003	0.	.0012	.0004	.0008	0.		
ETA^2				.0190	.0113	.0054	.0106	.0091	.0182	.0188	.0182		

Table A-2.2. Breakdown by Aircraft Code for All Items in Sample OC.L.

Analysis of Variance for Differences Between Aircraft Groups
For Sample OC.L.

SNJMB H033T
DATE 4-9-81
S.BI

CODE	WEAPON	WEAPON No.		Z_1	Z_2	Z_3	Z_4	Z_6	Z_8	Z_{10}	Z_{12}		No. OF FORECASTS
9999	COMMON	0		.08	.00	-.02	-.02	.05	.04	.00	.01		3019
1012	B52	1		.03	.09	.03	.00	.01	-.08	-.08	-.08		3716
3072	B57	2		.18	-.04	-.14	-.10	-.14	-.17	.06	-.14		208
9362	C118	3		.67	.15	-.04	.09	.04	.06	-.15	-.10		142
9652	C121	4		.01	-.10	-.07	-.02	-.00	-.02	-.31	-.20		126
4132	C123	5		.28	.06	-.01	-.09	.02	-.14	-.00	-.38		121
4002	C130	6		.02	.12	-.01	-.11	-.03	-.16	.00	-.04		937
1192	C135	7		-.04	.03	-.04	-.09	.02	-.14	.02	-.01		1228
4762	C141	8		-.04	.01	.01	-.06	-.02	.15	.04	-.04		1584
3922	F4	9		.05	.12	.12	.03	.03	.05	.16	.02		1208
4202	F5	10		-.16	-.48	-.11	-.02	-.61	-.25	-.16	-.25		48
3052	F100	11		.06	-.11	-.07	.09	-.06	-.15	-.17	-.25		463
2172	F101	12		-.08	-.07	.03	-.09	.01	-.12	-.08	-.13		732
2012	F102	13		.02	.07	.08	.11	-.05	-.13	-.13	-.17		475
2032	F104	14		-.66	-.70	-.50	-.66	-.75	-.75	-.63	-.74		649
3062	F105	15		-.12	-.06	-.02	-.11	-.13	-.16	-.12	-.06		689
2010	F106	16		-.12	-.04	-.06	-.03	-.00	-.19	-.14	-.03		1073
9682	T33	17		.16	.00	.11	.04	.00	-.00	.04	-.04		263
9242	T37	18		.22	-.03	.24	-.01	.43	-.16	-.12	.11		76
4202	T38	19		-.04	-.00	.09	.01	.01	-.02	.01	.05		566
4522	T39	20		-.03	-.15	-.09	-.00	-.05	-.03	-.22	-.36		181
3142	TF111	21		.08	.34	-.10	-.14	.17	.05	.20	.09		1329
9602	CT29	22		-.01	-.06	.04	-.07	-.00	.10	-.13	-.20		170
OVERALL MEAN				.0002	0.0272	-0.0184	-0.0529	-0.0115	-0.0612	-0.0345	-0.0682		14053
STD. DEV.				1.6931	1.1056	1.6824	1.6231	1.6839	1.4527	1.4988	1.4539		
ANOVA RESULTS													
BETWEEN GROUPS F				7.788	9.092	3.674	5.506	7.548	11.480	9.207	9.560		
" " SIGNIFICANCE				0.	0.	0.	0.	0.	0.	0.	0.		
R^2				0.0008	0.	0.0002	0.0005	0.0001	0.0003	0.	0.0001		
ETA^2				0.0089	0.0104	[illegible]	[illegible]	[illegible]	0.0131	[illegible]	[illegible]		

Table A-2.3. Breakdown by Aircraft Code for All Items in Sample SM.H.

Analysis of Variance for Differences Between Aircraft Groups
For Sample SM.H.

SNUMB A3681
DATE 4-4-81
S.B1

CODE	WEAPON	WEAPON N.	Z_1	Z_2	Z_3	Z_4	Z_6	Z_8	Z_{10}	Z_{12}	No. of FORECASTS
9999	COMMON	0	.13	-.04	-.21	-.19	-.31	-.11	-.13	-.21	672
1012	B52	1									
3072	B57	2									
7562	C118	3									
9552	C121	4	.01	-.19	-.09	-.20	-.14	-.29	-.47	-.42	84
7132	C123	5									
4202	C130	6									
1192	C135	7									
4762	C141	8									
3722	F4	9									
4203	F5	10									
3052	F100	11	-.08	-.11	-.11	-.10	-.45	-.38	-.50	-.59	360
2172	F101	12									
2012	F102	13									
3032	F104	14	-.81	-1.13	-1.14	-1.06	-.98	-.72	-1.07	-.83	224
3062	F105	15	-.25	-.20	-.37	-.17	-.35	-.16	-.31	-.30	392
2010	F106	16								-	
9682	T38	17	.11	-.12	.13	-.27	-.35	-.09	-.26	-.37	116
4242	T37	18									
4202	T38	19									
4522	T39	20	-.16	-.72	.05	-.27	-.41	-.48	-.67	-.38	228
3242	TF111	21	.10	-.20	-.01	-.10	-.26	-.11	-.17	-.06	~~444~~
4602	CT29	22									
OVERALL MEAN			-.0789	-.2754	-.2253	-.2624	-.3959	-.2532	-.3729	-.3372	2320
STD. DEV.			1.9310	1.9787	1.9051	1.9986	1.8253	1.7105	1.6961	1.8309	
ANOVA RESULTS											
BETWEEN GROUPS F			7.055	9.641	10.043	6.144	4.150	4.716	9.695	5.116	
" " SIGNIFICANCE			0.	0.	0.	0.	0.	0.	0.	0.	
r^2			.0018	.0046	.0004	.0001	.0002	.0008	.0025	0.	
ETA^2			.0093	.0262	.0272	.0168	.0114	.0190	[illegible]	.0141	

Table A-2.4. Breakdown by Aircraft Code for All Items in Sample SM.L.

Analysis of Variance for Differences Between Aircraft Groups
For Sample SM.L.

SNUMB A376T
DATE 4-4-81
S.B1

CODE	WEAPON	WEAPON No.	Z_1	Z_2	Z_3	Z_4	Z_6	Z_8	Z_{10}	Z_{12}	No. OF FORECASTS
9999	COMMON	0	.04	0.00	-.01	-.06	-.09	-.11	-.08	-.12	7412
1012	B52	1									
3072	C57	2									
7362	C118	3									
9652	C121	4	.07	-.06	.02	-.09	-.00	-.12	-.20	-.24	2472
4192	C123	5									
4002	C130	6									
1192	C135	7									
4762	C141	8									
3722	F4	9									
4262	F5	10									
3052	F100	11	.12	-.00	-.06	-.01	-.14	-.25	-.24	-.32	2920
2172	F101	12									
2012	F102	13									
3032	F104	14	-.49	-.56	-.64	-.61	-.64	-.50	-.53	-.58	2064
3062	F105	15	-.02	-.15	-.09	-.04	-.16	-.19	-.23	-.24	3792
2016	F106	16									[illegible]
9682	T39	17	.08	.06	.25	.07	.03	-.01	-.04	-.05	1600
9242	T37	18									
4202	T38	19									
4522	T34	20	.75	-.02	-.03	-.06	.14	-.17	-.25	-.23	1016
3292	TF111	21	.03	-.02	.07	.12	.05	.03	.13	.14	5128
9602	CT39	22									
OVERALL MEAN			.0069	-.0713	-.0344	-.0563	-.0981	-.1418	-.1321	-.1578	26404
STD. DEV.			1.7755	1.7006	1.776	1.7886	1.7088	1.5078	1.5174	1.5676	
ANOVA RESULTS											
BETWEEN GROUPS F			27.544	31.278	34.832	37.578	42.674	32.482	51.393	57.869	
" " SIGNIFICANCE			0.	0.	0.	0	0.	0.	0.	0.	
r^2			.0002	.0003	0.	.0004	.0002	.0002	.0005	.0009	
ETA^2			.0073	.0082	.0092	.0099	.0112	.0085	.0134	.0151	

Table A-3.1. Breakdown by Aircraft for Sample OC.H. Excluding F104 & F5 Items.

Analysis of Variance for Differences Between Aircraft Groups
For Sample OC.H.

SNUMB N324T/S63
DATE 4-16-81

CODE	WEAPON	WEAPON NC	Z_1	Z_2	Z_3	Z_4	Z_6	Z_5	Z_{11}	Z_{12}	NO. OF FORECASTS
9999	COMMON	0	.04	.07	-.02	-.01	.03	-.05	.01	-.03	2004
1012	B52	1	.22	.02	.05	-.08	.07	-.10	-.25	-.22	2284
3072	C5?	2	.07	.02	-.00	.21	-.22	-.48	-.09	.02	116
7362	C118	3	.65	.07	.02	.03	.22	-.00	.00	-.15	124
4692	C121	4	.61	-.26	.00	-.19	-.03	-.14	-.48	-.48	44
4132	C123	5	.16	.04	-.30	-.57	-.26	-.38	-.41	-.45	120
4002	C130	6	-.34	-.09	-.12	-.24	-.16	-.23	-.18	-.33	1076
1182	C135	7	-.10	.05	-.01	-.21	-.12	-.15	-.08	-.29	740
4762	C141	8	.14	.07	.02	.11	-.04	.14	.07	.09	720
3922	F4	9	.16	.39	.20	.16	.23	.12	.28	.15	1968
4202	F5	10									
3052	F100	11	-.05	.01	-.08	-.07	-.11	-.18	-.38	-.50	328
2172	F101	12	-.18	.05	-.13	-.04	-.07	-.22	-.25	-.27	260
2012	F102	13	.02	-.10	-.08	.26	-.01	-.25	-.40	-.30	180
9032	F104	14									
3062	F105	15	-.12	-.09	-.05	-.03	-.34	-.09	-.05	-.02	364
2016	F106	16	-.03	-.01	.08	.10	-.03	-.33	-.06	-.27	660
9682	T33	17	.24	.26	-.18	.01	-.20	.06	.11	-.07	216
9042	T37	18	.10	.24	-.11	.48	-.09	-.12	.20	.27	96
4202	T38	19	.13	-.01	-.09	-.28	-.43	-.16	-.18	-.14	288
4522	T39	20	.09	-.07	-.13	-.21	-.28	.17	-.37	-.29	100
3242	TF111	21	.04	.53	-.05	-.02	.07	-.11	.59	.13	480
4602	CT24	22	-.00	.01	-.13	-.06	.06	-.27	-.36	-.61	172
OVERALL MEAN			0.0591	0.0994	0.0122	-0.0283	-0.0069	-0.0845	-0.0362	-0.1183	12340
STD. DEV			1.8492	1.9909	1.9894	2.0137	2.1068	1.7641	1.8612	1.8094	
ANOVA RESULTS											
BETWEEN GROUPS F			5.719	4.471	1.835	3.912	3.574	4.188	10.105	7.157	
" " SIGNIFICANCE			0.	0.	0.0128	0.	0.	0.	0.	0.	
r^2			0.0003	0.0005	0.	0.0002	0.0008	0.0001	0.0014	0.0001	
ETA^2			0.0092	0.0072	0.0030	0.0063	0.0058	0.0068	0.0161	0.0115	

Table A-3.2. Breakdown by Aircraft for Sample OC.L. Excluding F104 & F5 Items.

Analysis of Variance for Differences Between Aircraft Groups
For Sampel OC.L.

SAJMB N325T/533
DATE 4-16-81

CODE	WEAPON	WEAPON No.	Z_1	Z_2	Z_3	Z_4	Z_6	Z_5	Z_{11}	Z_{12}	No. OF FORECASTS
9999	COMMON	0	.08	.00	-.02	-.02	.05	.04	.00	.001	3019
1012	652	1	.03	.09	.03	.00	.01	-.08	-.08	-.08	3716
3072	037	2	.18	-.09	-.14	-.10	-.14	-.17	.06	-.14	208
1562	C.18	3	.67	.15	-.04	.09	.04	.00	-.15	-.10	142
7652	C.121	4	.01	-.10	-.07	-.02	-.00	-.02	-.31	-.20	126
7132	C.123	5	.28	.06	-.01	-.09	.02	-.14	-.00	-.38	121
4002	C130	6	.02	.12	-.01	-.11	-.03	-.16	.00	-.04	937
1192	C.135	7	-.04	.03	-.04	-.04	.02	-.14	.02	-.01	1228
4762	C.141	8	-.04	.01	.01	-.06	-.02	.15	.04	-.04	1584
3922	F4	9	.05	.12	.12	.03	.03	.05	.16	.02	1208
7202	F5	10								-	.
3052	F100	11	.06	-.11	-.07	.09	-.06	-.15	-.17	-.25	463
2192	F.101	12	-.08	-.07	.02	-.09	.01	-.12	-.08	-.13	732
2012	F102	13	.08	.07	.08	.11	-.05	-.13	-.13	-.17	475
9032	F104	14									
3062	F105	15	-.12	-.06	-.02	-.11	-.13	-.16	-.12	-.06	689
2010	F106	16	-.12	-.04	-.06	-.03	-.00	-.19	-.14	-.03	1073
9682	T38	17	.16	.00	.11	.04	.00	-.00	.04	.04	263
9242	T37	18	.22	-.03	.24	-.01	.43	-.16	-.12	.11	76
9202	T38	19	-.04	-.00	.09	.01	.01	-.02	.01	.05	566
4522	T39	20	-.03	-.15	-.09	-.00	-.05	-.03	-.22	-.36	181
3242	TF111	21	.08	.34	-.10	-.17	.17	.05	.20	.09	1379
9602	C T39	22	-.01	-.06	.04	-.07	-.00	.10	-.12	-.20	170
OVERALL MEAN			0.0240	0.0542	-0.0011	-0.0314	0.0163	-0.0390	-0.0130	-0.0438	18356
STD. DEV.			1.6173	1.6893	1.6463	1.6055	1.6698	1.4327	1.4676	1.4310	
ANOVA RESULTS											
BETWEEN GROUPS F			3.164	3.699	1.323	1.344	1.513	4.964	4.920	3.260	
" " SIGNIFICANCE			0.	0.	0.1511	0.1389	0.0058	0.	0.	0.	
r^2			0.0003	0.0001	0.	0.0002	0.	0.	0.0002	0.	
ETA^2			0.0034	0.0040	0.0014	0.0015	[illegible]	[illegible]	[illegible]	[illegible]	

Table A-3.3. Breakdown by Aircraft for Sample SM.H Excluding F104 & F5 Items.

Analysis of Variance for Differences Between Aircraft Group
For Sample SM.H.

SNUMB N178T / S.83
DATE 4-16-81

CODE	WEAPON	WEAPON No.		Z_1	Z_2	Z_3	Z_4	Z_6	[illegible]	Z_{10}	Z_{12}		No. OF FORECASTS
9999	COMMON	0		.13	-.04	-.21	-.19	-.31	-.11	-.13	-.21		672
1012	B52	1											
3072	C97	2											
1362	C118	3											
4552	C121	4		.01	-.19	-.09	-.20	-.14	-.29	-.47	-.42		84
4132	C123	5											
4002	C130	6											
1192	C135	7											
4762	C141	8											
3722	F4	9											
4202	F5	10	— EXCLUDED										
3052	F100	11		-.08	-.11	-.11	-.10	-.45	-.38	-.50	-.57		360
2172	F101	12											
2012	F102	13											
3032	F104	14	— EXCLUDED										
3062	F105	15		-.25	-.20	-.37	-.17	-.35	-.16	-.31	-.30		392
2010	F106	16											
9682	T38	17		.11	-.12	.13	-.27	-.35	-.09	-.26	-.37		116
4242	T37	18											
4202	T38	19											
4522	T39	20		-.16	-.72	.05	-.27	-.41	-.48	-.67	-.38		228
3242	TF111	21		.10	-.20	-.01	-.10	-.26	-.11	-.17	-.06		444
9602	CT29	22											
OVERALL MEAN				-0.0072	-0.1903	-0.1360	-0.1726	-0.3389	-0.2067	-0.3049	-0.2886		2296
STD. DEV.				1.7856	1.8557	1.7571	1.8409	1.6819	1.5694	1.5434	1.6126		
ANOVA RESULTS													
BETWEEN GROUPS F				2.757	3.952	2.064	0.370	0.719	2.756	5.289	3.813		
" " SIGNIFICANCE				0.0113	0.0006	0.0140	0.8980	0.6344	0.0113	0.	0.0009		
r^2				0.0013	0.0038	0.0015	0	0.	0.0025	0.0020	0.0001		
ETA^2				0.0072	0.0103	0.0069	0.0010	0.0019	0.0072	0.0037	0.0099		

Table A-3.4. Breakdown by Aircraft for Sample SM.L Excluding F104 & F5 Items.

Analysis of Variance for Differences Between Aircraft Groups
For Sample SM.L.

SNUMB N3237 S.B3
DATE 7-16-81

CODE	WEAPON	WEAPON No.		Z_1	Z_2	Z_3	Z_4	Z_6	[illegible]	Z_{11}	Z_{12}		No. OF FORECASTS
9999	COMMON	0		.04	.00	-.08	-.06	-.09	-.11	-.08	-.12		7412
1012	B52	1											
3072	B57	2											
1562	C118	3											
9652	C121	4		.07	-.06	.02	-.09	-.00	-.12	-.20	-.24		2472
4132	C123	5											
4002	C130	6											
1192	C135	7											
4762	C141	8											
5922	F4	9											
4202	F5	10	— EXCLUDED										
3052	F100	11		.12	-.00	-.06	-.01	-.14	-.25	-.24	-.32		2920
2172	F101	12											
2012	F102	13											
9032	F104	14	— EXCLUDED										
3662	F105	15		-.02	-.15	-.09	-.04	-.16	-.19	-.23	-.24		3792
2016	F106	16											
9682	T38	17		.08	.06	.25	.07	.03	-.01	-.04	-.05		1600
4242	T37	18											
4202	T38	19											
4522	T39	20		.07	-.02	-.03	-.06	.14	-.17	-.25	-.23		1016
3242	TF111	21		.03	-.02	.07	.12	.05	.03	.13	.14		5128
9602	CT29	22											
OVERALL MEAN				.0193	-.0297	.0098	-.0090	-.0517	-.1106	-.0982	-.1230		24340
STD. DEV				1.6987	1.6221	1.7036	1.7313	1.6362	1.4117	1.4332	1.4866		
ANOVA RESULTS													
BETWEEN GROUPS F				2.766	5.859	10.353	8.044	12.069	17.867	32.301	44.905		
" " SIGNIFICANCE				0.0109	0.	0.	0.	0.	0.	0.	0.		
R^2				0.	0.0001	0.0002	0.0012	0.0007	0.0006	0.0011	0.0019		
ETA^2				0.0007	0.0014	0.0026	0.0020	0.0030	[illegible]	0.0098	0.0110		

Table A-4.1. Breakdown by Base Period for Sample OC.H Excluding F104 & F5 Items.

Analysis of Variance for Differences Between Forecast Base Periods.
For Sample OC.H.

SNUMB N3241/S.B3 DATE 4-16-81	PERIOD NO	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_{11}	Z_{12}	No. of FORECASTS
BASE PERIOD										
	1	.06	.00	-.08	-.17	-.36	-.46	-0.27	-.28	3085
	2	.19	.07	.10	.11	.10	.16	.11	.08	3085
	3	-.00	.12	-.03	.05	.18	-.03	.00	-.28	3085
	4	-.00	.18	.06	-.10	.04	0.	0.	0.	3085
OVERALL MEAN		0.0591	0.0094	0.0122	-0.0283	-0.0069	-0.0845	-0.0362	-0.1183	12340
" STD DEV		1.8547	1.9956	1.9895	2.0152	2.1007	1.7537	1.8495	1.8110	
ANOVA RESULTS										
BETWEEN GROUPS F		8.047	4.557	6.106	14.134	42.193	71.409	24.727	34.699	
" SIGNIFICANCE		0.	0.0035	0.0004	0.	0.	0.	0	0	
r^2		0.0006	0.0011	0.0003	0.0001	0.0048	0.0057	0.0048	0.0009	
ETA^2		0.0020	0.0011	0.0015	0.0034	0.0102	0.0171	0.0060	0.0084	

Table A-4.2. Breakdown by Base Period for Sample OC.L Excluding F104 & F5 Items.

Analysis of Variance for Differences Between Forecast Base Periods
For Sample OC.L.

SNUMB 11325T/5.B3 DATE 4-16-81	PERIOD No	Z_1	Z_2	Z_3	Z_4	Z_6	Z_8	Z_{10}	Z_{12}	No. of Forecasts
BASE PERIOD										
	1	.09	.06	.03	-.07	-.08	-.07	-.07	-.11	4583
	2	.10	.07	-.00	-.03	.03	-.09	.01	.00	4568
	3	-.09	-.02	-.05	-.02	.05	-.07	.00	-.06	4538
	4	-.01	.10	.02	.00	.06	0	0.	0.	4667
OVERALL MEAN		0.0240	0.0542	-0.0011	-0.0314	0.0163	-0.0390	-0.0130	-0.0438	18 356
" STD DEV		1.6172	1.6912	1.6464	1.6057	1.6693	1.4355	1.4705	1.4320	
ANOVA RESULTS										
BETWEEN GROUPS F		16.768	5.159	2.302	1.799	8.130	3.595	3.556	7.176	
" " SIGNIFICANCE		0.	0.0015	0.0753	0.1453	0.	0.0131	0.0138	0.0001	
r^2		0.0015	0.	0.	0.0003	0.0010	0.0001	0.0002	0.0004	
ETA^2		0.0027	0.0008	0.0004	0.0003	0.0015	0.0006	0.0006	0.0012	

Table A-4.3. Breakdown by Base Period for Sample SM.H Excluding F104 & F5 Items.

Analysis of Variance for Differences Between Forecast Base Periods For Sample SM.H.

SNUMB N178T/S.83 DATE 4-16-81	Period No		Z_1	Z_2	Z_3	Z_4	Z_6	Z_8	Z_{10}	Z_{12}		No. of Forecasts
BASE PERIOD												
	1		-.17	-.40	-.26	-.42	-.57	-.41	-.53	-.43		574
	2		.17	-.06	-.01	.05	-.26	-.11	-.34	-.20		574
	3		-.03	-.37	-.18	-.17	-.22	-.30	-.33	-.51		574
	4		.00	.07	-.07	-.13	-.29	0.	0.	0.		574
OVERALL MEAN			-0.0072	-0.1983	-0.1360	-0.1736	-0.3387	-0.2067	-0.3049	-0.2888		2296
" STD DEV			1.7662	1.8528	1.7593	1.8327	1.6764	1.5657	1.5409	1.6068		
ANOVA RESULTS												
BETWEEN GROUPS F			3.944	9.256	2.368	6.546	5.030	9.965	12.067	12.121		
" " SIGNIFICANCE			0.0081	0.	0.0690	0.0002	0.0018	0.	0.	0.		
r^2			0.0044	0.0046	0.0006	0.0019	0.0033	0.0056	0.0136	0.0047		
ETA^2			0.0051	0.0120	0.0031	0.0085	0.0065	0.0103	0.0155	0.0156		

Table A-5.1. Breakdown by Dollar Demand Class for All Items in Sample OC.H.

Analysis of Variance for Differences Between Dollar Demand Classes For Sample OC.H.

Group LD907-S.B2
Date 4-13-81

Class Upper Bound	Class No.		Z_1	Z_2	Z_3	Z_4	Z_6	Z_8	Z_{10}	Z_{12}		No. of Fore-Casts
-.01	-1		1.23	.95	.97	1.36	1.24	1.09	.62	.65		221
.10	-0		.10	.05	.17	.12	.13	.11	.09	.06		391
.30	1											
1.00	2											
3.00	3		2.15	3.29	3.29	4.81	3.29	2.56	2.33	2.33		4
10.00	4		.60	-.29	.37	1.24	.53	-0.06	-.01	-.06		15
31.00	5		1.16	1.05	1.73	1.34	1.22	.56	.25	.88		67
100.00	6		.68	.80	.57	.55	1.19	.97	.66	.48		190
316.00	7		.56	.58	.47	.67	.94	.39	.46	.33		574
1000.00	8		.11	.21	.21	.16	.28	.09	.11	.10		1814
3162.00	9		-.04	-.01	-.11	-.15	-.15	-.20	-.13	-.21		4789
10000.00	10		-.05	-.03	-.12	-.23	-.26	-.25	-.18	-.28		3189
Hi	11		-.12	-.00	-.19	-.30	-.35	-.32	-.16	-.35		1362
Overall Mean			0.0390	0.0799	-0.0627	-0.0482	-0.0255	-0.1060	-0.0626	-0.1355		12616
Standard Deviation			1.8673	2.0097	2.0041	2.0115	2.1017	1.7634	1.8867	1.8277		
ANOVA Results												
Between Groups F			22.427	16.202	22.731	39.609	43.641	30.888	15.241	21.34		
Between Groups Significance			0	0	0	0	0.	0.	0	0		
r^2			0.0086	0.0045	0.0087	0.0142	0.0156	0.0143	0.0055	0.0091		
ETA^2			0.075	0.0127	0.0177	0.0267	0.0335	0.0254	0.0119	0.0166		

Table A-5.2. Breakdown by Dollar Demand Class for All Items in Sample OC.L.

Analysis of Variance for Differences Between Dollar Demand Classes
For Sample OC.L.

Smurb L 09 IT-SO2 Date 4-13-81 Class Upper Bound	Class No.		Z_1	Z_2	Z_3	Z_4	Z_6	Z_8	Z_{10}	Z_{12}		No. of Fore-Casts
-.01	-1		.70	.69	.77	.70	.78	.45	.99	.56		799
.10	-0		.03	.04	.04	.04	.03	.21	.02	.02		3488
.30	1		.25	.15	-.01	-.09	.08	-.18	-.03	-.19		134
1.00	2		-.13	.06	.03	-.15	-.02	-.08	-.07	-.20		491
3.10	3		-.00	.02	.01	.00	-.15	-.08	-.14	-.01		1183
10.00	4		.05	-.00	-.08	-.04	-.01	-.01	-.07	-.12		1991
31.00	5		-.00	-.02	-.03	-.05	.01	-.07	-.01	-.09		2648
100.00	6		-.02	.03	0.	-.04	.00	-.06	.01	-.06		3061
316.00	7		-.09	-.01	.11	-.22	-.09	-.22	-.10	-.21		2847
1000.00	8		-.09	-.05	-.12	-.21	-.21	-.17	-.17	-.13		1917
3162.00	9		-.37	-.23	-.49	-.34	-.12	-.17	-.25	-.30		495
10000.00	10		-.39	-.42	-.79	-.52	-.77	-.23	-.18	-.77		69
HI	11		-.64	-.03	-.18	.10	-.46	-.10	.23	.32		10
Overall Mean			0.0002	0.0272	-0.0184	-0.0529	-0.0145	-0.0647	-0.0345	-0.0682		19053
Standard Deviation			1.6412	1.7075	1.6792	1.6166	1.6801	1.4556	1.5012	1.4525		
ANOVA Results												
Between Groups F			17.086	12.322	21.656	22.115	20.200	13.847	11.016	19.870		
Between Groups Significance			0.	0.	0.	0	0	0	0	0		
r^2			0.0043	0.0025	0.0058	0.0075	0.0047	0.0050	0.0029	0.0058		
ETA^2			0.0106	0.0077	0.0135	0.0138	0.0126	0.0086	0.0069	0.0124		

Table A-5.3. Breakdown by Dollar Demand Class for All Items in Sample SM.H.

Analysis of Variance for Differences Between Dollar Demand Classes For Sample SM.H.

Sample LD221-S.B2
Date 4-13-81

Class Upper Bound	Class No.		z_1	z_2	z_3	z_4	z_6	z_8	z_{10}	z_{12}		No. of Forecasts
-.01	-1		.75	.98	.63	.85	1.09	.66	.29	.55		55
.10	-0		.05	.02	.06	.01	.01	.06	.01	.01		108
.30	1											
1.00	2											
3.10	3		-.34	4.72	-0.34	-.34	-.34	0.	0.	0.		2
10.00	4		2.98	2.98	-.51	-.51	2.98	-.15	3.333	-0.15		3
31.[illegible]	5		1.03	.95	.05	.09	-.25	-.02	.23	-.07		11
100.00	6		.46	.46	.62	.46	.63	.27	.01	.38		60
316.00	7		.18	.10	-.00	.30	.16	0.00	-.20	.00		172
1000.00	8		.03	-.06	-.11	.02	-.21	-.08	-.17	-.26		550
3162.[illegible]	9		-.16	-.41	-.36	-.49	-.59	-.34	-.49	-.49		1040
10000.00	10		-.27	-.58	-.39	-.45	-.76	-.51	-.59	-.44		412
[illegible]1	11		-.58	-1.09	-.39	-.67	-.77	-.71	-.79	-.60		107
Overall Mean			-0.0789	-0.2254	-0.2253	-0.2524	-0.3959	-0.2532	0.3229	-0.3372		2520
Standard Deviation			1.9336	1.9652	1.9180	1.9865	1.7904	1.7040	1.6985	1.8286		
ANOVA Results												
Between Groups F			4.560	10.596	3.873	7.728	13.220	5.562	6.070	4.521		
Between Groups Significance			0.	0.	0.	0.	0	0.	0.	0.		
r^2			0.0085	0.0214	0.0092	0.0147	0.0301	0.0148	0.0135	0.0116		
ETA^2			0.0178	0.0405	0.0152	0.0299	0.0501	0.0217	0.0236	0.0177		

Table A-5.4. Breakdown by Dollar Demand Class for All Items in Sample SM.L.

Analysis of Variance for Differences Between Dollar Demand Classes
For Sample SM.L.

Smmb L088T-SB2 Date 4-13-81 Class Upper Bound	Class No.		Z_1	Z_2	Z_3	Z_4	Z_6	Z_8	Z_{10}	Z_{12}		No. of Fore-Casts
-.01	-1		.83	.87	.82	.91	.83	.56	.73	.64		742
.10	-0		.03	.03	.03	.03	.03	.01	.02	.01		6944
.30	1		.87	.36	.84	.33	.18	.42	.07	-.10		132
1.00	2		.38	.04	.10	.04	-.06	-.21	-.12	-.19		609
3.10	3		.04	-.05	.09	.00	-.04	-.11	-.07	-.16		1528
10.00	4		.07	-.04	.06	.00	-.07	-.19	-.14	-.17		316
31.00	5		.09	-.04	-.03	-.05	-.06	-.12	-.17	-.24		4062
100.00	6		-.03	-.14	-.14	-.44	-.19	-.27	-.22	-.26		4076
316.00	7		-.21	-.29	-.23	-.25	-.35	-.30	-.34	-.31		3202
1000.00	8		-.29	-.39	-.35	-.36	-.43	-.36	-.36	-.35		1692
3162.00	9		-.64	-.64	-.71	-.68	-.59	-.57	-.64	-.67		339
10000.00	10		.02	-.46	-.95	-.59	-.68	-.32	-.14	-.66		19
Hi	11		1.33	-.70	.59	-1.36	4.46	.21	.21	-.56		2
Overall Mean			0.0069	-0.0713	-0.0349	0.0563	-0.0981	-0.148	-0.1324	-0.1538		26404
Standard Deviation			1.7689	1.6938	1.7664	1.7844	1.6956	1.5032	1.5142	1.5676		
ANOVA Results												
Between Groups F			33.070	34.498	33.769	32.953	39.111	32.990	32.940	31.218		
Between Groups Significance			0	0	0	0	0	0	0	0		
r^2			0.0059	0.0094	0.0079	0.0081	0.0108	0.0107	0.0126	0.0114		
ETA^2			0.0148	0.0163	0.0151	0.0148	0.0125	0.0148	0.0178	0.0153		

Appendix B

Empirical Distribution of Period Errors Zi Excluding F104 and F5 Items for Samples OC.L, SM.H, and SM.L

Cumulative Probabilities for Period Errors Z for 1971-72 Base Year Forecasts With General Program Factors for Sample OC.L

PERCENTAGES

		1	2	3	4	5	6	7	8	9	10	11	12
1	-9.0	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.
2	-8.0	0.	0.	0.	1.	0.	0.	0.	1.	0.	0.	0.	0.
3	-7.0	0.	0.	0.	1.	0.	0.	0.	1.	0.	0.	0.	1.
4	-6.0	0.	0.	0.	1.	0.	0.	0.	1.	0.	0.	0.	1.
5	-5.0	0.	0.	0.	1.	1.	1.	1.	1.	1.	1.	1.	1.
6	-4.0	0.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.
7	-3.0	1.	1.	1.	2.	1.	1.	1.	2.	1.	1.	2.	2.
8	-2.0	2.	2.	3.	4.	3.	3.	3.	4.	3.	3.	3.	4.
9	-1.0	12.	12.	12.	14.	13.	14.	13.	15.	14.	14.	14.	15.
10	0.	68.	68.	67.	70.	70.	71.	70.	71.	68.	71.	71.	72.
11	1.0	83.	83.	83.	84.	85.	85.	84.	84.	82.	84.	85.	86.
12	2.0	89.	89.	88.	89.	90.	90.	89.	89.	87.	89.	89.	90.
13	3.0	93.	92.	91.	92.	93.	92.	92.	92.	91.	91.	92.	92.
14	4.0	94.	94.	93.	94.	95.	94.	94.	93.	92.	93.	93.	94.
15	5.0	96.	96.	95.	96.	96.	95.	95.	95.	94.	94.	95.	95.
16	6.0	97.	96.	96.	96.	96.	96.	96.	96.	95.	95.	95.	96.
17	7.0	97.	97.	97.	97.	97.	97.	96.	97.	96.	96.	96.	96.
18	8.0	98.	97.	97.	97.	97.	97.	97.	97.	96.	96.	96.	97.
19	9.0	98.	98.	98.	98.	98.	98.	97.	98.	97.	97.	97.	97.
20	10.0	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.
TOTAL	OBS	16083	16083	16083	16083	16083	16083	16083	16083	16083	16083	16083	16083

TABLE	MEAN	VARIANCE	C.OF V.	SKEWNESS	KURTOSIS
1	0.281	9.404	10.918	6.363	72.236
2	0.386	12.636	9.198	6.331	61.669
3	0.444	14.704	8.633	5.754	53.897
4	0.263	17.843	16.086	3.522	52.326
5	0.288	15.378	13.636	5.212	57.881
6	0.383	17.346	10.879	5.773	53.138
7	0.404	17.092	10.242	5.213	50.944
8	0.315	18.164	13.516	4.239	48.996
9	0.609	21.606	7.633	5.040	40.762
10	0.593	23.124	8.113	5.375	41.530
11	0.453	20.429	9.986	5.040	46.714
12	0.354	20.405	12.776	4.715	46.754

Cumulative Probabilities for Period Errors Z
for 1973-74 Base Year Forecasts With General Program Factors
for Sample OC.L

PERCENTAGES

		1	2	3	4	5	6	7	8	9	10	11	12
1	-9.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
2	-8.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
3	-7.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
4	-6.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
5	-5.0	0.	0.	1.	0.	0.	0.	0.	0.	0.	1.	1.	0.
6	-4.0	1.	1.	1.	1.	0.	1.	0.	0.	0.	1.	1.	0.
7	-3.0	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.
8	-2.0	3.	3.	3.	3.	2.	3.	2.	2.	2.	3.	2.	2.
9	-1.0	13.	13.	14.	14.	13.	13.	13.	13.	14.	13.	14.	13.
10	0.	65.	67.	68.	70.	69.	69.	71.	71.	72.	69.	71.	71.
11	1.0	80.	82.	83.	84.	83.	83.	84.	85.	85.	83.	85.	85.
12	2.0	87.	88.	89.	89.	88.	89.	89.	89.	90.	88.	89.	89.
13	3.0	91.	91.	92.	92.	91.	91.	92.	92.	93.	91.	92.	92.
14	4.0	93.	93.	94.	94.	93.	93.	94.	94.	94.	93.	94.	93.
15	5.0	95.	95.	95.	95.	95.	95.	95.	95.	95.	95.	95.	95.
16	6.0	96.	95.	96.	96.	95.	96.	96.	96.	96.	96.	96.	96.
17	7.0	96.	96.	96.	97.	96.	96.	97.	97.	97.	96.	96.	96.
18	8.0	97.	97.	97.	97.	97.	97.	97.	97.	97.	97.	97.	97.
19	9.0	97.	97.	98.	98.	97.	97.	98.	98.	98.	97.	98.	97.
20	10.0	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.
TOTAL	OBS	1338	1338	1338	1338	1338	1338	1338	1338	1338	1338	1338	1338

TABLE	MEAN	VARIANCE	C.OF V.	SKEWNESS	KURTOSIS
13	0.592	16.200	6.804	5.742	50.396
14	0.541	16.499	7.513	6.003	50.895
15	0.444	15.913	8.988	5.911	54.802
16	0.390	15.294	10.040	5.894	54.162
17	0.544	18.965	7.998	5.186	47.200
18	0.463	17.321	8.988	5.372	50.819
19	0.427	16.503	9.516	6.016	54.931
20	0.424	15.606	9.324	6.286	56.961
21	0.308	14.127	12.191	5.165	59.653
22	0.408	18.230	10.472	4.069	48.568
23	0.300	16.502	13.536	3.804	54.264
24	0.414	16.875	9.922	4.785	53.379

Cumulative Probabilities for Period Errors Z for 1975-76 Base Year Forecasts With General Program Factors for Sample OC.L

PERCENTAGES

		1	2	3	4	5	6	7	8	9	10	11	12
1	-9.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
2	-8.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
3	-7.0	0.	0.	0.	0.	1.	0.	1.	0.	1.	0.	0.	0.
4	-6.0	0.	0.	1.	0.	1.	1.	1.	1.	1.	0.	1.	1.
5	-5.0	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.
6	-4.0	1.	1.	2.	1.	2.	2.	2.	2.	2.	2.	2.	2.
7	-3.0	3.	3.	4.	3.	4.	3.	4.	3.	4.	3.	4.	3.
8	-2.0	4.	5.	5.	4.	6.	5.	5.	5.	5.	5.	5.	5.
9	-1.0	17.	16.	17.	16.	18.	16.	18.	18.	19.	18.	18.	19.
10	0.	72.	69.	72.	71.	71.	68.	71.	74.	75.	71.	72.	74.
11	1.0	86.	84.	85.	85.	84.	82.	84.	86.	87.	84.	85.	86.
12	2.0	91.	90.	90.	90.	90.	88.	90.	91.	91.	90.	90.	91.
13	3.0	94.	93.	93.	93.	93.	91.	92.	93.	94.	93.	92.	93.
14	4.0	96.	95.	95.	95.	95.	93.	94.	95.	95.	95.	94.	95.
15	5.0	97.	96.	96.	96.	96.	95.	96.	96.	96.	96.	96.	95.
16	6.0	98.	97.	97.	97.	97.	96.	96.	97.	97.	96.	96.	97.
17	7.0	98.	97.	98.	97.	97.	96.	97.	97.	97.	97.	97.	97.
18	8.0	98.	98.	98.	98.	98.	97.	97.	98.	98.	97.	97.	97.
19	9.0	98.	98.	98.	98.	98.	97.	98.	98.	98.	98.	98.	98.
20	10.0	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.

TOTAL OBS125541255412554125541255412554125541255412554125541255412554

TABLE	MEAN	VARIANCE	C.OF V.	SKEWNESS	KURTOSIS
25	0.054	9.579	56.789	6.365	72.037
26	0.147	11.857	23.420	4.801	65.509
27	0.041	11.522	83.549	4.089	60.454
28	0.154	12.687	23.075	4.480	61.925
29	0.093	14.978	41.408	4.024	58.543
30	0.353	17.094	11.704	3.892	49.652
31	0.189	16.056	21.170	5.041	53.968
32	0.080	13.746	46.129	5.235	60.304
33	-0.006	13.153	594.796	4.283	60.859
34	0.210	15.696	18.855	4.942	55.917
35	0.211	16.748	19.378	4.992	50.471
36	0.076	14.333	49.663	4.489	59.304

Cumulative Probabilities for Period Errors Z for 1977-78 Base Year Forecasts With General Program Factors for Sample OC.L

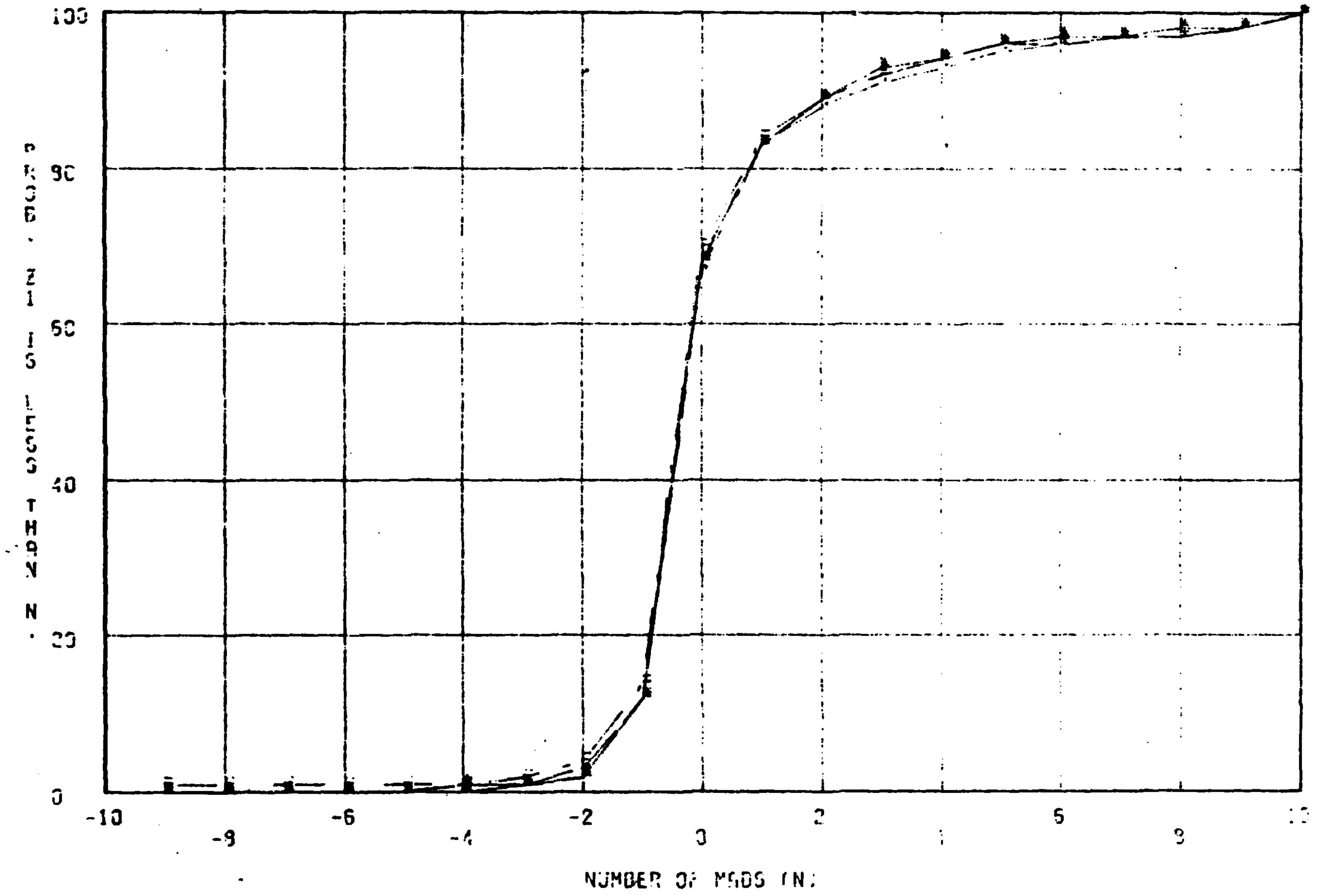
PERCENTAGES

		1	2	3	4	5	6	7	8	9	10	11	12
1	-9.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
2	-8.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
3	-7.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
4	-6.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
5	-5.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
6	-4.0	1.	0.	1.	1.	0.	0.	0.	0.	0.	0.	0.	0.
7	-3.0	1.	1.	1.	1.	1.	1.	0.	0.	0.	0.	0.	0.
8	-2.0	3.	2.	3.	3.	2.	2.	0.	0.	0.	0.	0.	0.
9	-1.0	15.	13.	14.	15.	14.	14.	0.	0.	0.	0.	0.	0.
10	0.	66.	63.	64.	66.	65.	64.	0.	0.	0.	0.	0.	0.
11	1.0	85.	82.	83.	84.	83.	82.	0.	0.	0.	0.	0.	0.
12	2.0	91.	88.	88.	90.	89.	88.	0.	0.	0.	0.	0.	0.
13	3.0	94.	92.	92.	93.	92.	91.	0.	0.	0.	0.	0.	0.
14	4.0	95.	94.	94.	95.	94.	93.	0.	0.	0.	0.	0.	0.
15	5.0	97.	96.	96.	96.	96.	95.	0.	0.	0.	0.	0.	0.
16	6.0	97.	97.	97.	97.	97.	96.	0.	0.	0.	0.	0.	0.
17	7.0	98.	97.	97.	97.	97.	97.	0.	0.	0.	0.	0.	0.
18	8.0	98.	98.	98.	98.	97.	97.	0.	0.	0.	0.	0.	0.
19	9.0	99.	98.	98.	98.	98.	98.	0.	0.	0.	0.	0.	0.
20	10.0	100.	100.	100.	100.	100.	100.	0.	0.	0.	0.	0.	0.
TOTAL	OBS	11639	11639	11639	11639	11639	11639	0	0	0	0	0	0

TABLE	MEAN	VARIANCE	C.OF V.	SKEWNESS	KURTOSIS
37	0.118	9.145	25.519	4.404	76.822
38	0.390	10.683	8.373	6.327	66.276
39	0.325	12.031	10.676	5.349	64.267
40	0.229	12.040	15.178	4.436	66.012
41	0.384	12.144	9.084	6.236	61.019
42	0.519	15.149	7.499	5.799	53.425

Z1 TO Z1 FOR 1971-1972 SSSL

FOR OF-1



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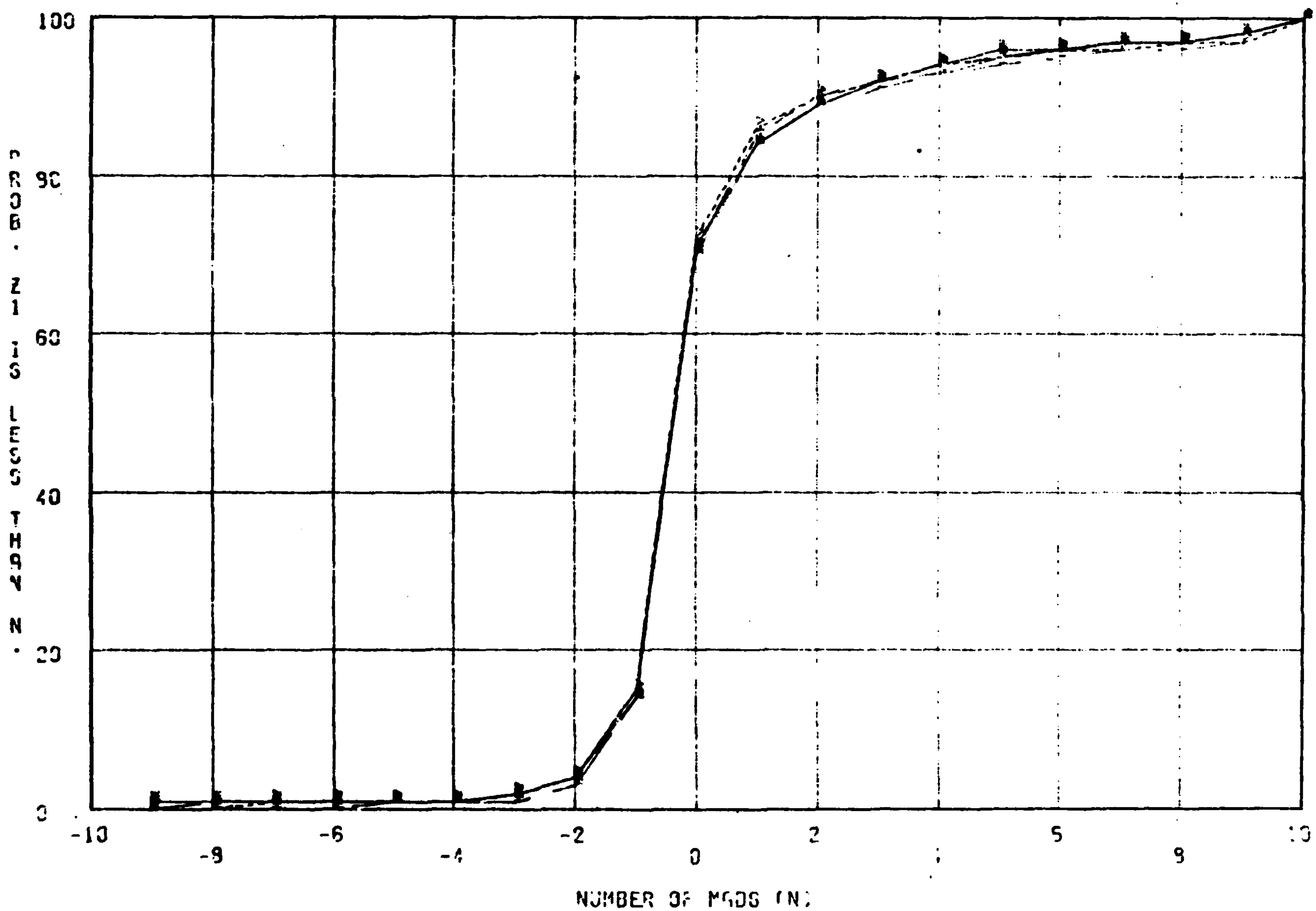
FOR OC-1



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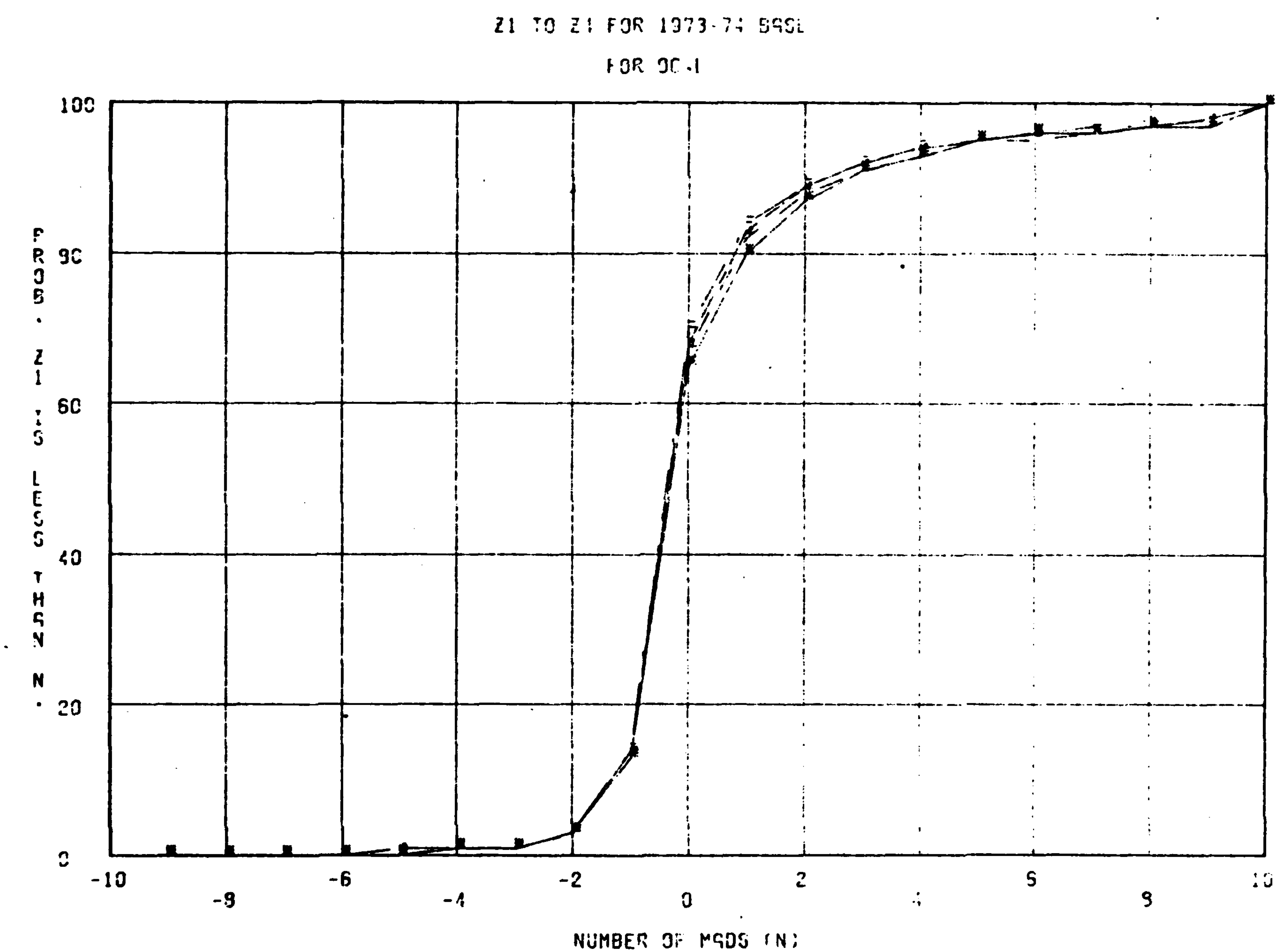
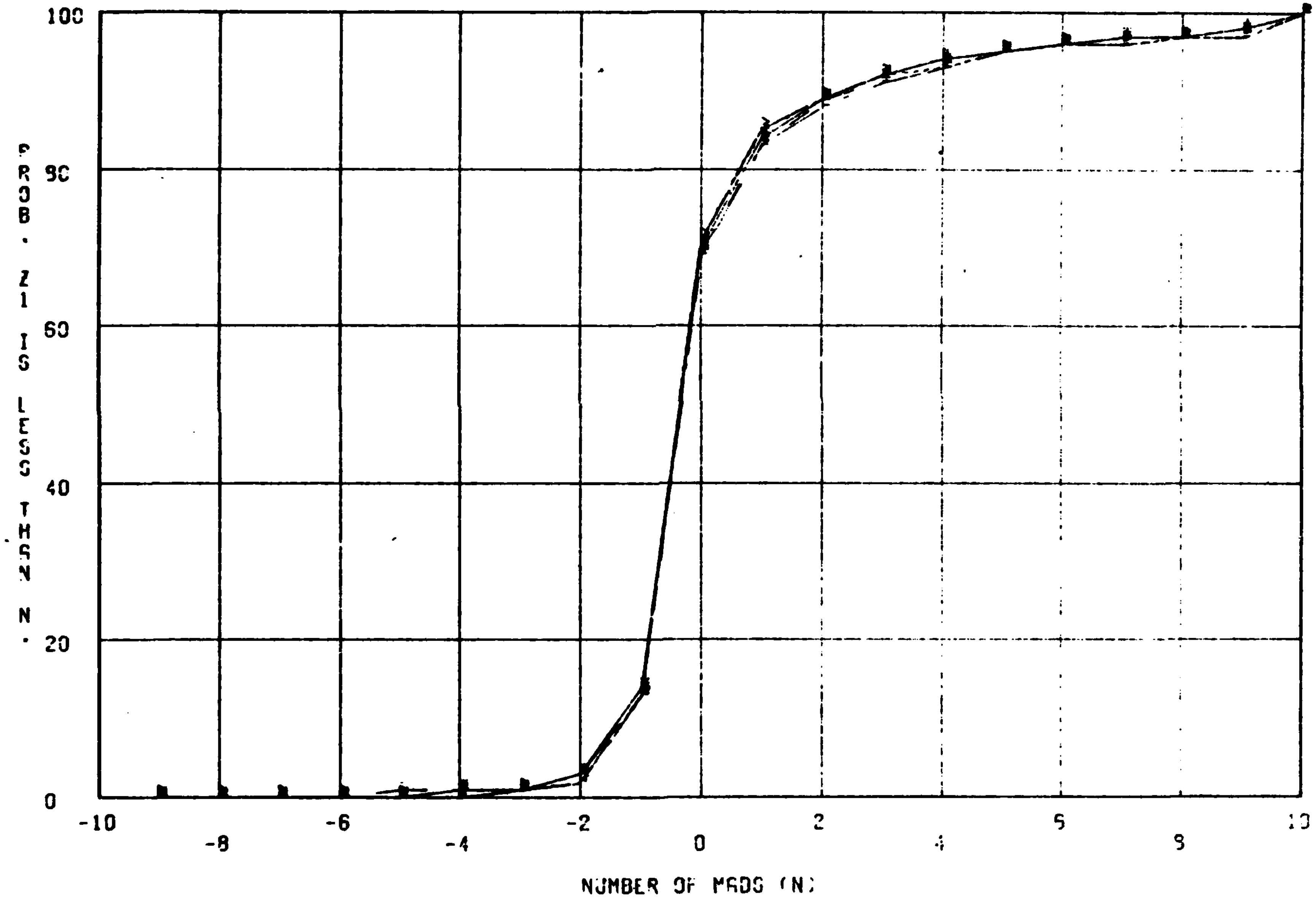


B-7

#3
μ006T
4-9-81

Z4. Z6. Z9. Z10. Z12 FOR 1973-1974 SSSL

FOR OC-I



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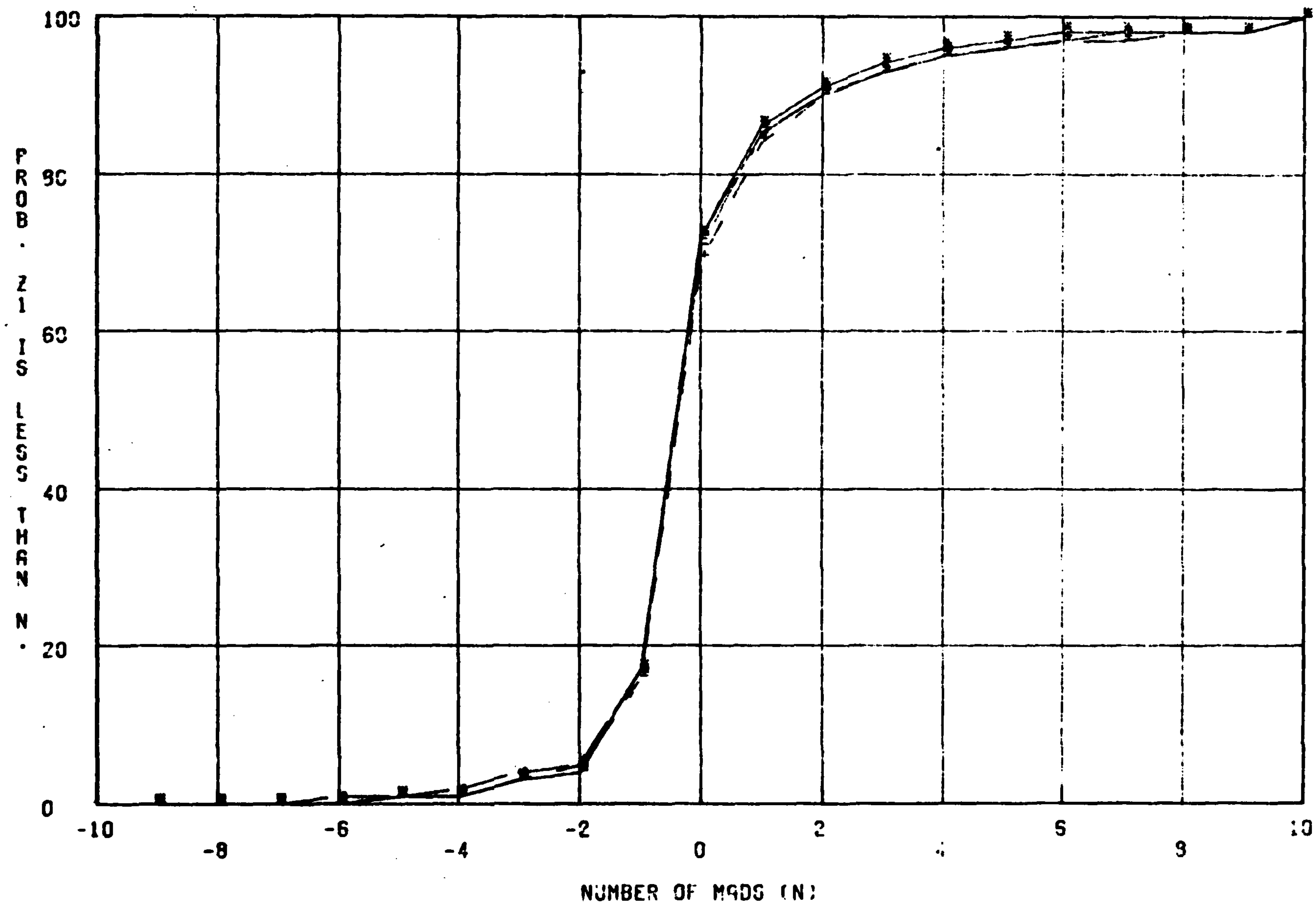
Z1 TO Z4 FOR 1975-76 BASE

FOR OC-1



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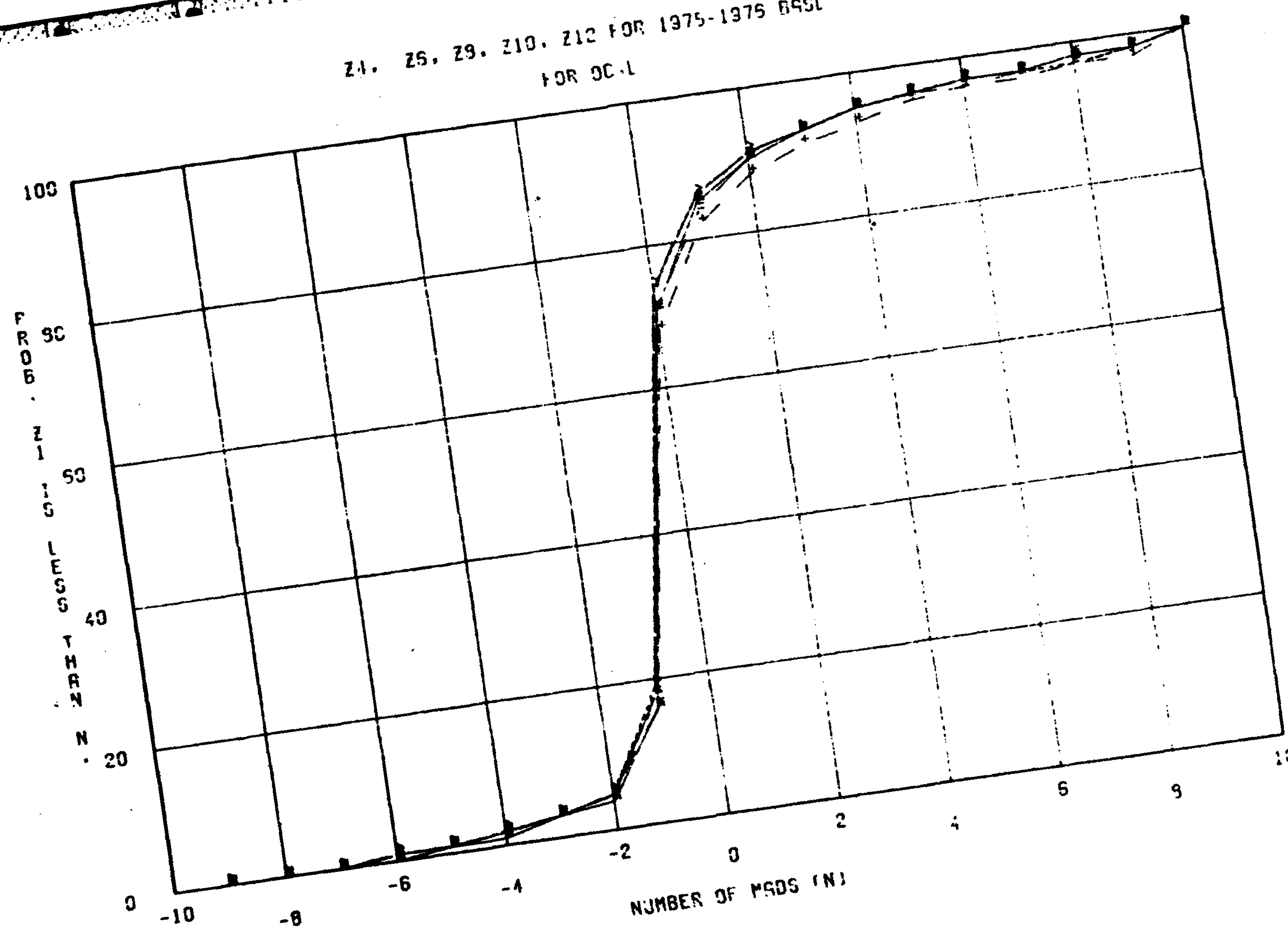
Z1, Z5, Z9, Z10, Z12 FOR 1975-1975 BASE

FOR OC-1

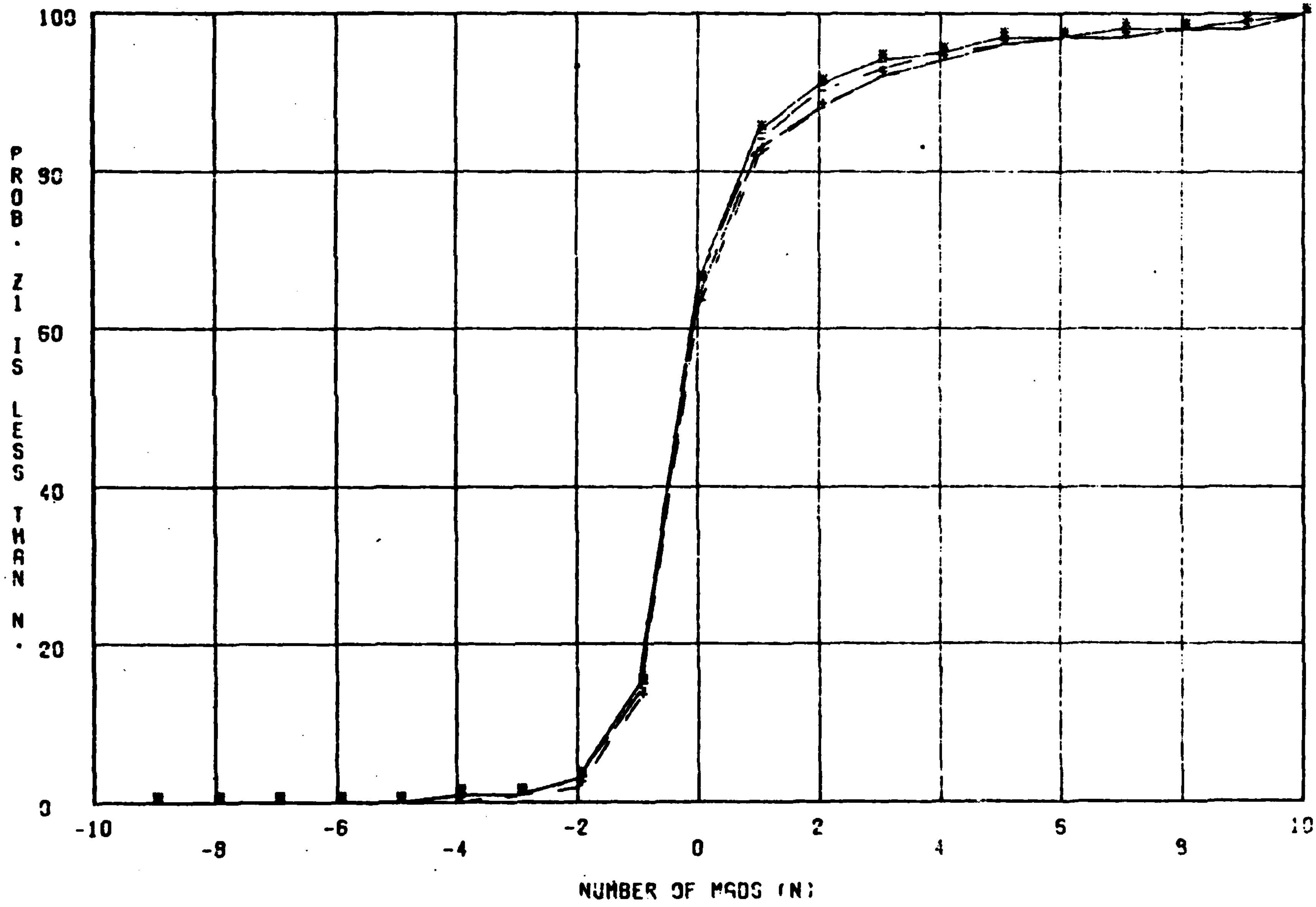


B-10

HG
HOOGT
4-9-81

Z1 TO Z4 FOR 1977-79 BASE

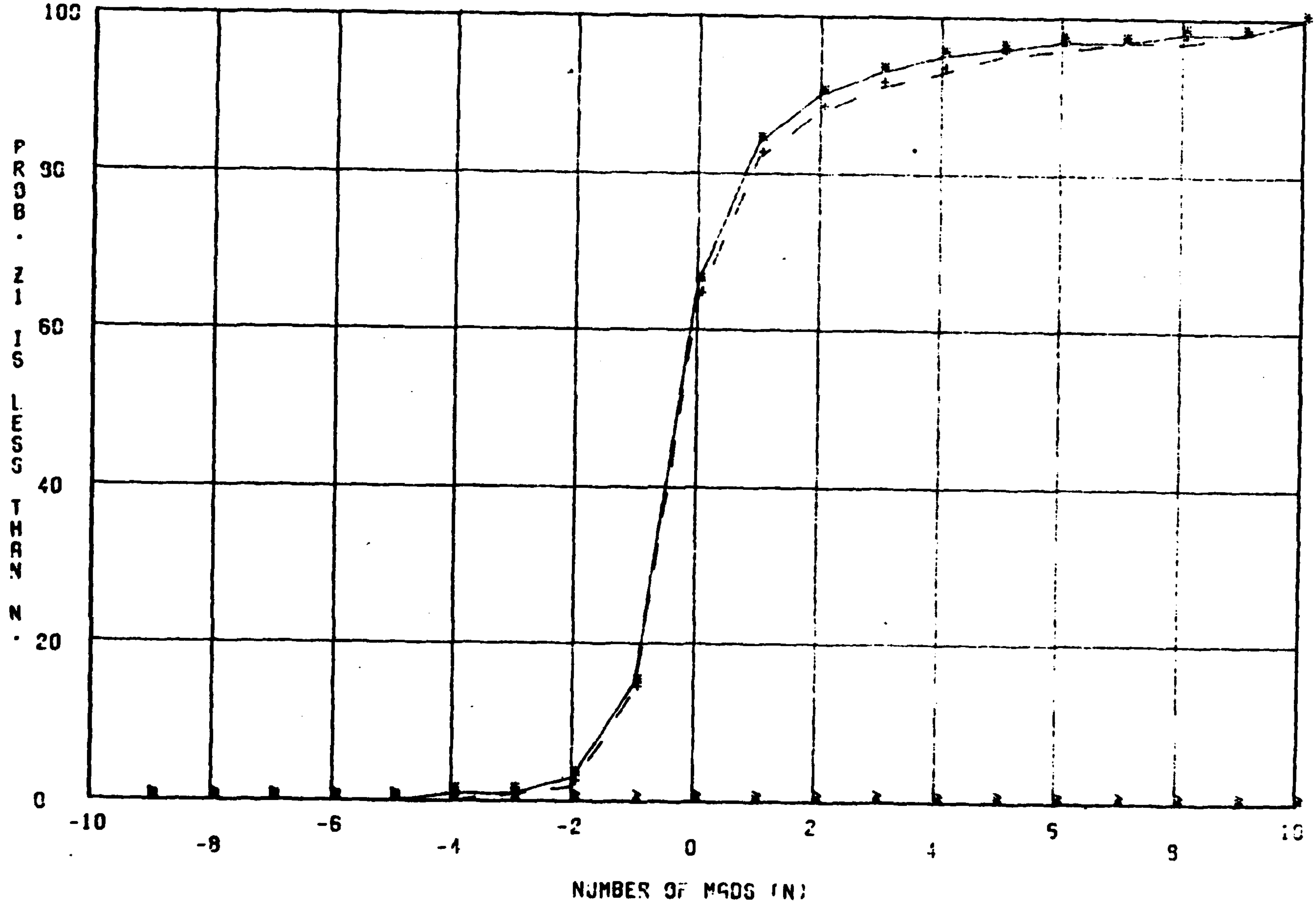
FOR OC-1



#7

Z4. Z6. Z8. Z10. Z12 FOR 1977-1979 DASL

FOR DC-1



Cumulative Probabilities for Period Errors Z for 1971-72 Base Year Forecasts With General Program Factors for Sample SM.H

PERCENTAGES

		1	2	3	4	5	6	7	8	9	10	11	12
1	-9.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0
2	-8.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0
3	-7.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0
4	-6.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0
5	-5.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0
6	-4.0	0.	1.	0.	2.	0.	0.	0.	1.	1.	0.	1.	1
7	-3.0	1.	2.	2.	3.	2.	1.	3.	2.	2.	2.	2.	2
8	-2.0	6.	8.	7.	7.	6.	5.	9.	7.	6.	7.	8.	9
9	-1.0	27.	31.	32.	33.	30.	34.	39.	34.	31.	33.	34.	33
10	0.	68.	72.	67.	73.	70.	78.	76.	73.	72.	77.	76.	78
11	1.0	85.	87.	84.	88.	85.	91.	89.	86.	86.	90.	88.	88
12	2.0	93.	95.	91.	93.	93.	97.	95.	93.	91.	95.	93.	93
13	3.0	96.	97.	96.	97.	96.	99.	97.	96.	96.	97.	97.	95
14	4.0	97.	98.	98.	98.	98.	99.	99.	97.	98.	98.	97.	96
15	5.0	98.	99.	99.	99.	99.	99.	99.	99.	98.	99.	99.	97
16	6.0	99.	99.	100.	100.	100.	100.	99.	100.	99.	99.	99.	98
17	7.0	99.	99.	100.	100.	100.	100.	100.	100.	99.	100.	99.	99
18	8.0	99.	99.	100.	100.	100.	100.	100.	100.	100.	100.	100.	99
19	9.0	100.	99.	100.	100.	100.	100.	100.	100.	100.	100.	100.	99
20	10.0	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100
TOTAL OBS		630	630	630	630	630	630	630	630	630	630	630	630

TABLE	MEAN	VARIANCE	C.OF V.	SKEWNESS	KURTOSIS
1	-0.177	2.815	9.466	2.492	15.604
2	-0.314	5.350	7.358	9.360	152.447
3	-0.288	2.586	5.590	1.441	8.487
4	-0.428	2.504	3.702	1.329	9.908
5	-0.280	2.506	5.644	2.339	15.832
6	-0.546	1.499	2.244	1.641	10.354
7	-0.567	2.659	2.875	3.183	31.745
8	-0.380	2.384	4.065	1.413	7.493
9	-0.279	3.141	6.358	3.055	24.031
10	-0.504	2.070	2.856	1.835	10.030
11	-0.421	3.329	4.335	3.841	38.847
12	-0.358	3.623	5.311	2.771	15.916

Cumulative Probabilities for Period Errors Z for 1973-74 Base Year Forecasts With General Program Factors for Sample SM.H

PERCENTAGES

		1	2	3	4	5	6	7	8	9	10	11	12
1	-9.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0
2	-8.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0
3	-7.0	0.	0.	0.	0.	0.	0.	1.	1.	0.	1.	1.	1
4	-6.0	0.	0.	0.	0.	0.	1.	1.	1.	0.	1.	1.	1
5	-5.0	0.	0.	0.	0.	0.	1.	1.	1.	0.	1.	1.	1
6	-4.0	0.	0.	0.	1.	0.	1.	1.	1.	0.	2.	2.	2
7	-3.0	1.	1.	1.	1.	1.	2.	2.	2.	1.	3.	3.	2
8	-2.0	4.	5.	4.	5.	5.	5.	6.	5.	4.	8.	6.	6
9	-1.0	23.	26.	25.	24.	29.	27.	26.	25.	25.	29.	28.	25
10	0.	58.	64.	64.	62.	67.	70.	66.	66.	69.	70.	68.	67
11	1.0	77.	82.	82.	79.	82.	85.	82.	82.	84.	84.	82.	[illegible]
12	2.0	86.	91.	89.	89.	91.	92.	90.	89.	90.	92.	89.	89
13	3.0	92.	95.	92.	93.	94.	95.	93.	93.	93.	94.	94.	93
14	4.0	95.	96.	95.	95.	95.	96.	95.	95.	95.	96.	96.	95
15	5.0	96.	98.	97.	96.	97.	98.	96.	96.	97.	98.	97.	96
16	6.0	97.	98.	97.	96.	97.	98.	97.	97.	97.	98.	98.	97
17	7.0	98.	98.	98.	97.	98.	98.	98.	98.	98.	99.	98.	98
18	8.0	98.	98.	98.	98.	98.	98.	98.	98.	98.	99.	98.	98
19	9.0	98.	99.	99.	98.	98.	99.	99.	99.	99.	99.	99.	99
20	10.0	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100
TOTAL OBS		622	622	622	622	622	622	622	622	622	622	622	62

TABLE	MEAN	VARIANCE	C.OF V.	SKEWNESS	KURTOSIS
13	0.350	7.611	7.880	4.362	30.824
14	0.016	5.250	140.590	4.675	39.690
15	0.122	6.639	21.110	4.469	42.678
16	0.334	11.746	10.273	5.124	37.139
17	0.023	8.242	122.717	6.256	71.585
18	-0.134	8.624	21.843	4.690	70.378
19	-0.065	13.886	57.279	-0.866	70.323
20	0.046	12.007	75.746	2.022	71.751
21	0.020	6.573	129.905	5.121	49.187
22	-0.325	13.632	11.359	-1.638	66.895
23	-0.005	10.045	653.076	5.964	62.880
24	-0.031	8.447	95.193	4.117	43.214

Cumulative Probabilities for Period Errors Z
for 1975-76 Base Year Forecasts With General Program Factor
for Sample SM.H

PERCENTAGES

		1	2	3	4	5	6	7	8	9	10	11	12
1	-9.0	1.	2.	1.	1.	0.	0.	0.	0.	0.	1.	1.	0
2	-8.0	1.	3.	1.	1.	1.	1.	1.	0.	1.	1.	1.	1
3	-7.0	2.	4.	2.	2.	1.	1.	1.	1.	1.	2.	2.	2
4	-6.0	2.	5.	3.	3.	2.	2.	2.	2.	2.	3.	2.	3
5	-5.0	3.	6.	4.	4.	3.	3.	3.	3.	3.	3.	4.	4
6	-4.0	4.	8.	6.	6.	5.	4.	5.	4.	5.	5.	6.	6
7	-3.0	8.	12.	10.	10.	9.	8.	10.	8.	9.	10.	10.	11
8	-2.0	12.	16.	14.	14.	14.	15.	17.	17.	18.	18.	18.	19
9	-1.0	29.	33.	31.	33.	33.	35.	36.	37.	39.	36.	39.	41
10	0.	69.	71.	71.	71.	71.	72.	72.	71.	76.	73.	77.	77
11	1.0	85.	86.	84.	86.	87.	86.	86.	87.	87.	89.	90.	90
12	2.0	93.	93.	91.	92.	92.	93.	91.	93.	92.	93.	94.	94
13	3.0	95.	96.	95.	94.	95.	95.	95.	95.	95.	96.	96.	97
14	4.0	97.	97.	97.	97.	97.	97.	97.	97.	97.	97.	98.	97
15	5.0	98.	99.	99.	98.	98.	98.	98.	98.	97.	98.	99.	99
16	6.0	98.	99.	99.	98.	99.	99.	99.	99.	98.	99.	99.	99
17	7.0	99.	99.	100.	99.	99.	99.	99.	99.	99.	99.	99.	99
18	8.0	99.	100.	100.	99.	99.	99.	99.	100.	99.	99.	99.	99
19	9.0	100.	100.	100.	99.	100.	100.	99.	100.	99.	99.	100.	99
20	10.0	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.	100
TOTAL OBS		601	601	601	601	601	601	601	601	601	601	601	60

TABLE	MEAN	VARIANCE	C.OF V.	SKEWNESS	KURTOSIS
25	-0.451	5.875	5.375	0.478	15.317
26	-0.908	15.718	4.367	-2.112	54.284
27	-0.643	6.056	3.828	-1.468	13.266
28	-0.591	7.316	4.573	0.242	18.034
29	-0.562	4.984	3.976	0.443	10.073
30	-0.596	4.977	3.744	0.326	9.643
31	-0.575	8.603	5.101	4.638	67.516
32	-0.611	5.651	3.889	2.056	25.157
33	-0.614	8.524	4.755	5.228	68.387
34	-0.643	10.386	5.010	5.404	70.236
35	-0.872	5.173	2.608	-0.557	14.977
36	-0.898	6.570	2.853	1.631	32.015

Cumulative Probabilities for Period Errors Z for 1977-78 Base Year Forecasts With General Program Factors for Sample SM.H

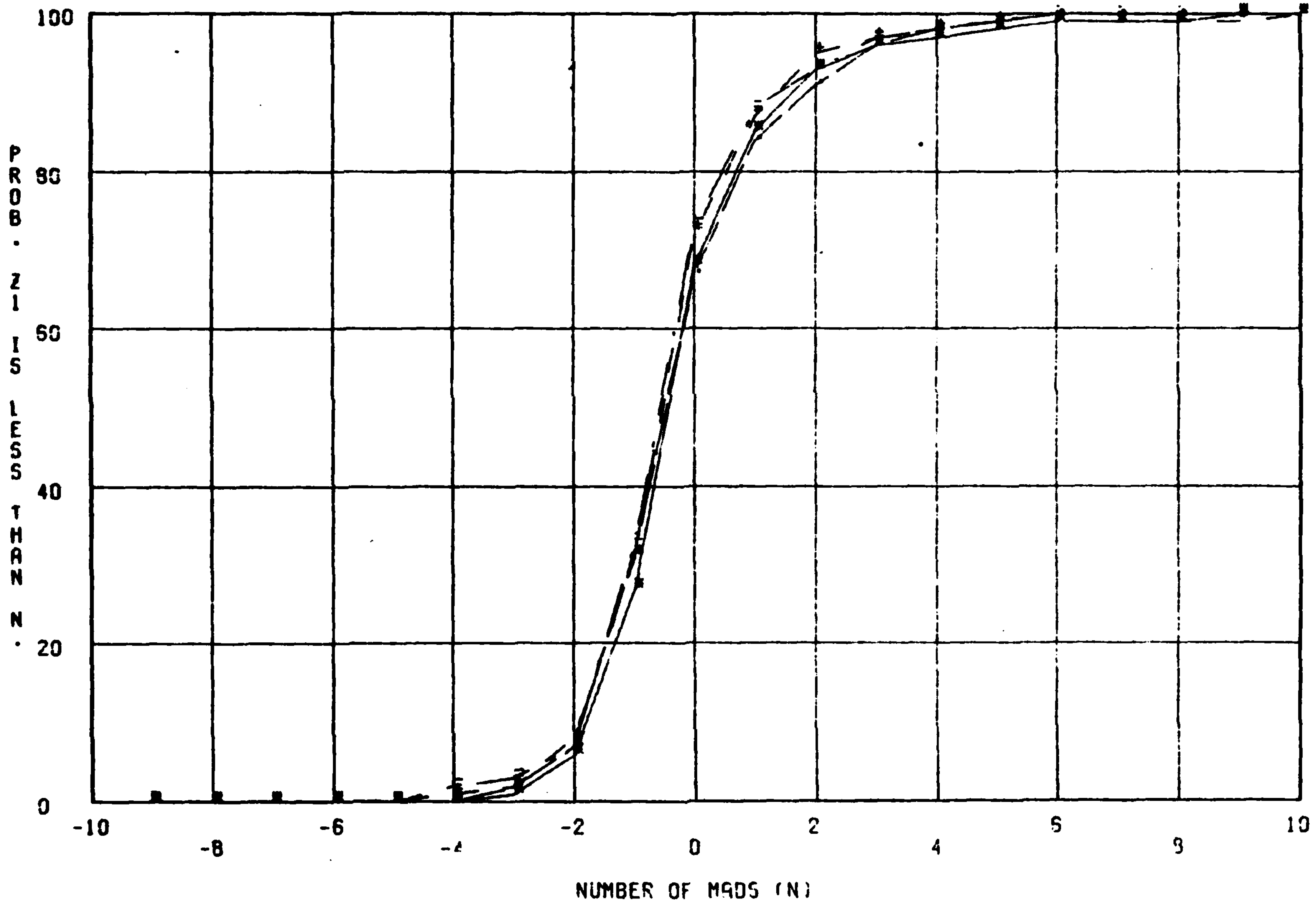
PERCENTAGES

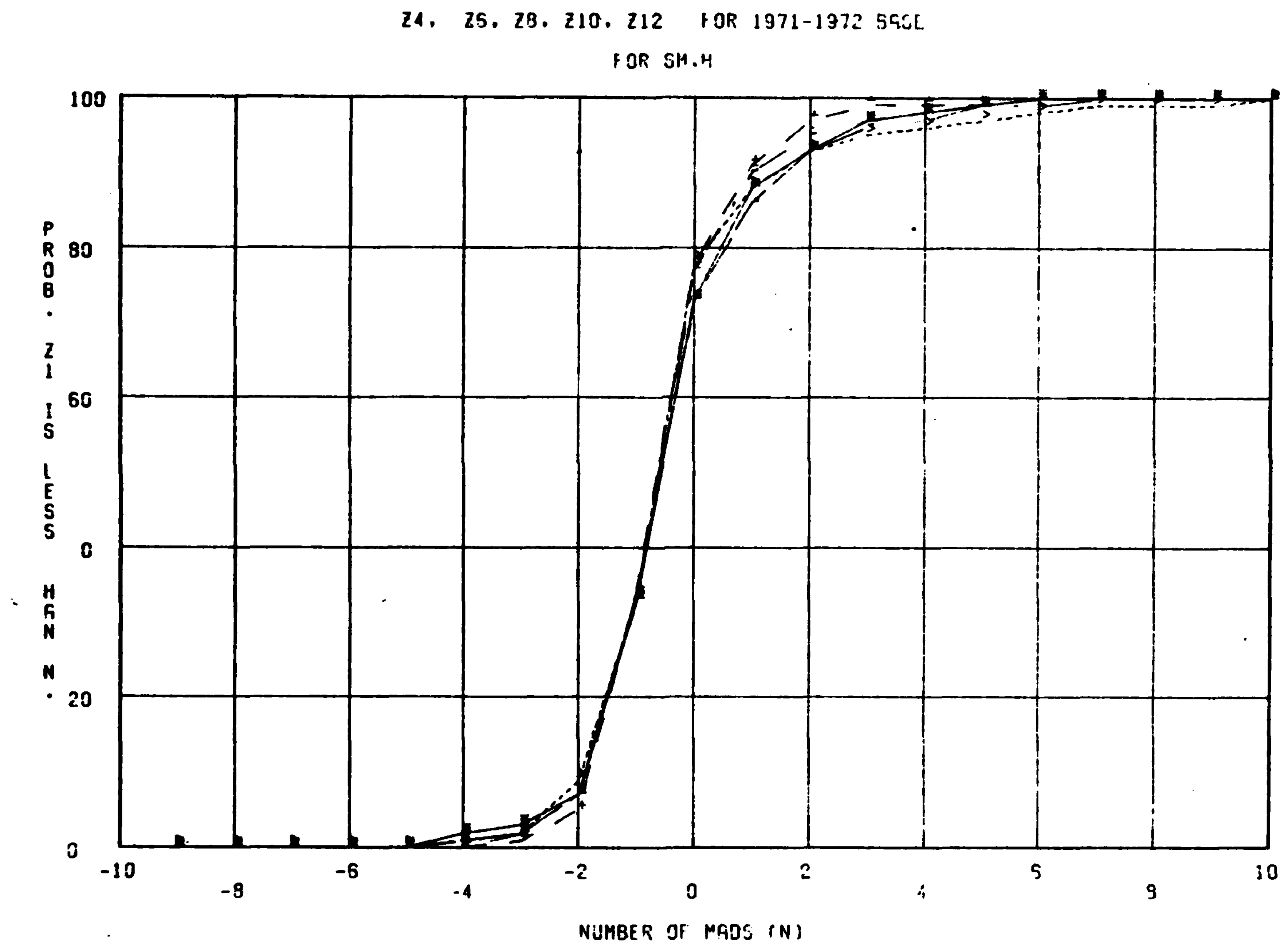
		1	2	3	4	5	6	7	8	9	10	11	12
1	-9.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0
2	-8.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0
3	-7.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0
4	-6.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0
5	-5.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0
6	-4.0	0.	0.	1.	1.	0.	1.	0.	0.	0.	0.	0.	0
7	-3.0	1.	1.	1.	3.	1.	3.	0.	0.	0.	0.	0.	0
8	-2.0	4.	4.	5.	7.	5.	10.	0.	0.	0.	0.	0.	0
9	-1.0	20.	22.	25.	20.	21.	31.	0.	0.	0.	0.	0.	0
10	0.	60.	60.	64.	64.	59.	70.	0.	0.	0.	0.	0.	0
11	1.0	81.	81.	85.	86.	78.	86.	0.	0.	0.	0.	0.	0
12	2.0	89.	90.	92.	93.	87.	92.	0.	0.	0.	0.	0.	0
13	3.0	94.	95.	95.	96.	92.	95.	0.	0.	0.	0.	0.	0
14	4.0	96.	96.	97.	97.	95.	96.	0.	0.	0.	0.	0.	0
15	5.0	98.	97.	98.	98.	96.	98.	0.	0.	0.	0.	0.	0
16	6.0	99.	98.	99.	98.	97.	98.	0.	0.	0.	0.	0.	0
17	7.0	99.	98.	99.	98.	98.	98.	0.	0.	0.	0.	0.	0
18	8.0	99.	99.	99.	99.	98.	99.	0.	0.	0.	0.	0.	0
19	9.0	99.	99.	99.	99.	99.	99.	0.	0.	0.	0.	0.	0
20	10.0	100.	100.	100.	100.	100.	100.	0.	0.	0.	0.	0.	0
TOTAL OBS		565	565	565	565	565	565	0	0	0	0	0	

TABLE	MEAN	VARIANCE	C.OF V.	SKEWNESS	KURTOSIS
37	0.134	5.265	17.130	5.179	48.905
38	0.156	6.683	16.533	6.640	71.357
39	-0.021	6.393	122.280	8.505	119.728
40	-0.058	7.584	47.393	7.828	96.397
41	0.263	7.070	10.093	6.905	92.182
42	-0.130	11.226	25.774	7.744	83.715

Z1 TO Z4 FOR 1971-1972 BASE

FOR SM.H

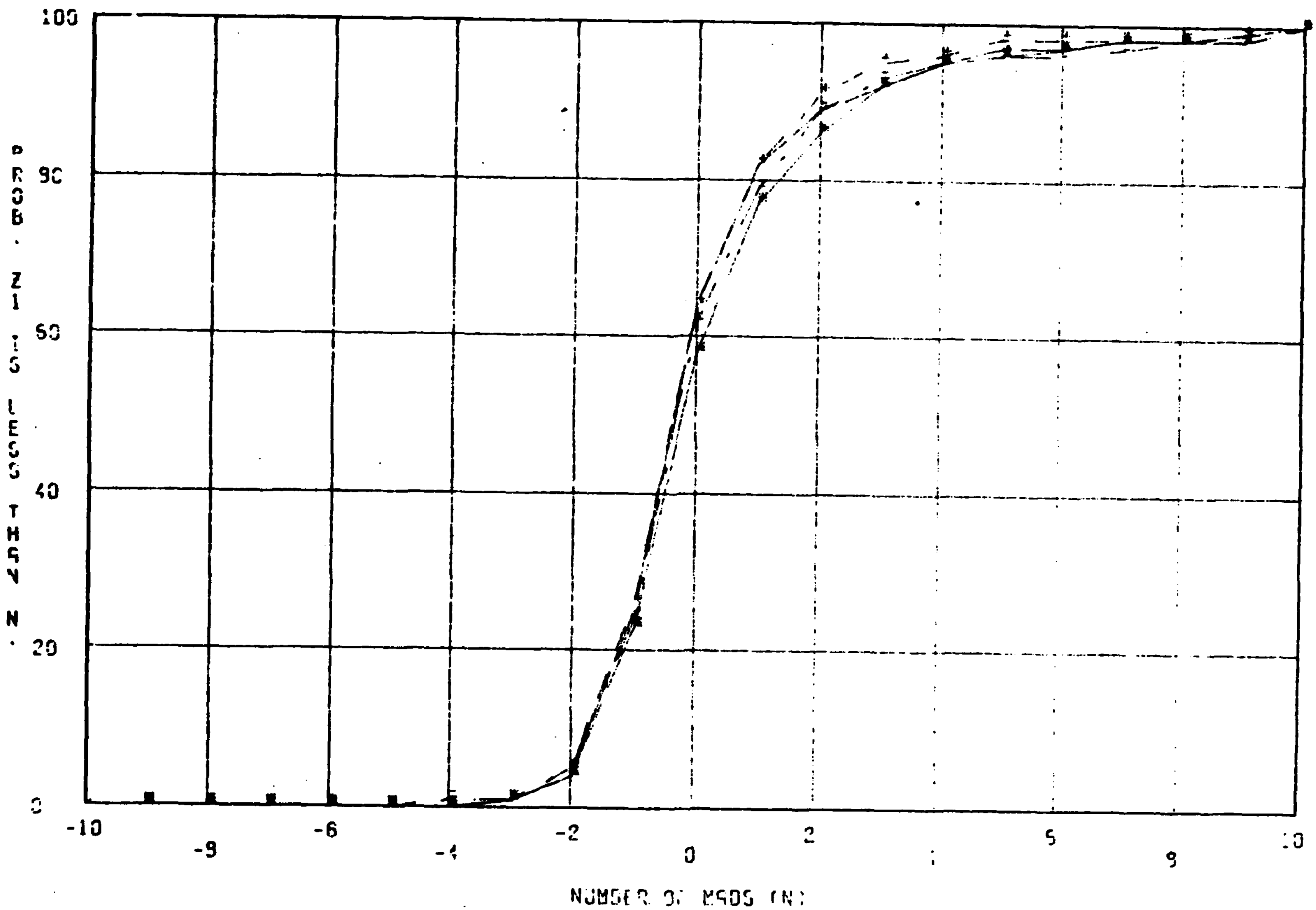




Z4, Z6, Z8, Z10, Z12 FOR 1971-1972 SSGL
FOR SM.H
100
80
60
0
20
0
PROB. Z1 IS LESS HAN N.
-10
-8
-6
-4
-2
0
2
4
5
9
10
NUMBER OF MADS (N)

Z1 TO Z1 FOR 1973-74 DSGE

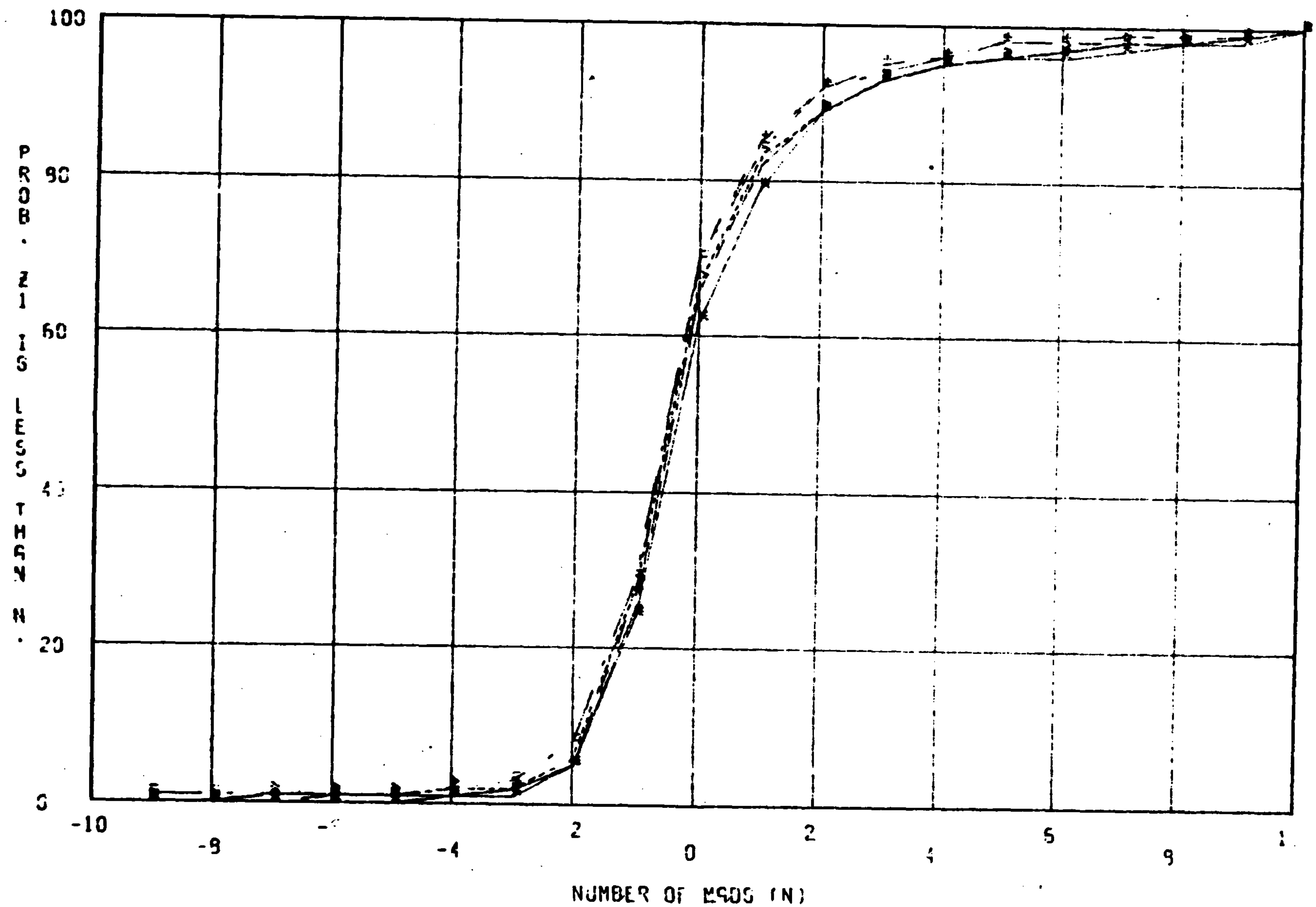
FOR SM.H



#3
H008T

Z-- Z5. Z9. Z10. Z12 FOR 973-1974 DASL

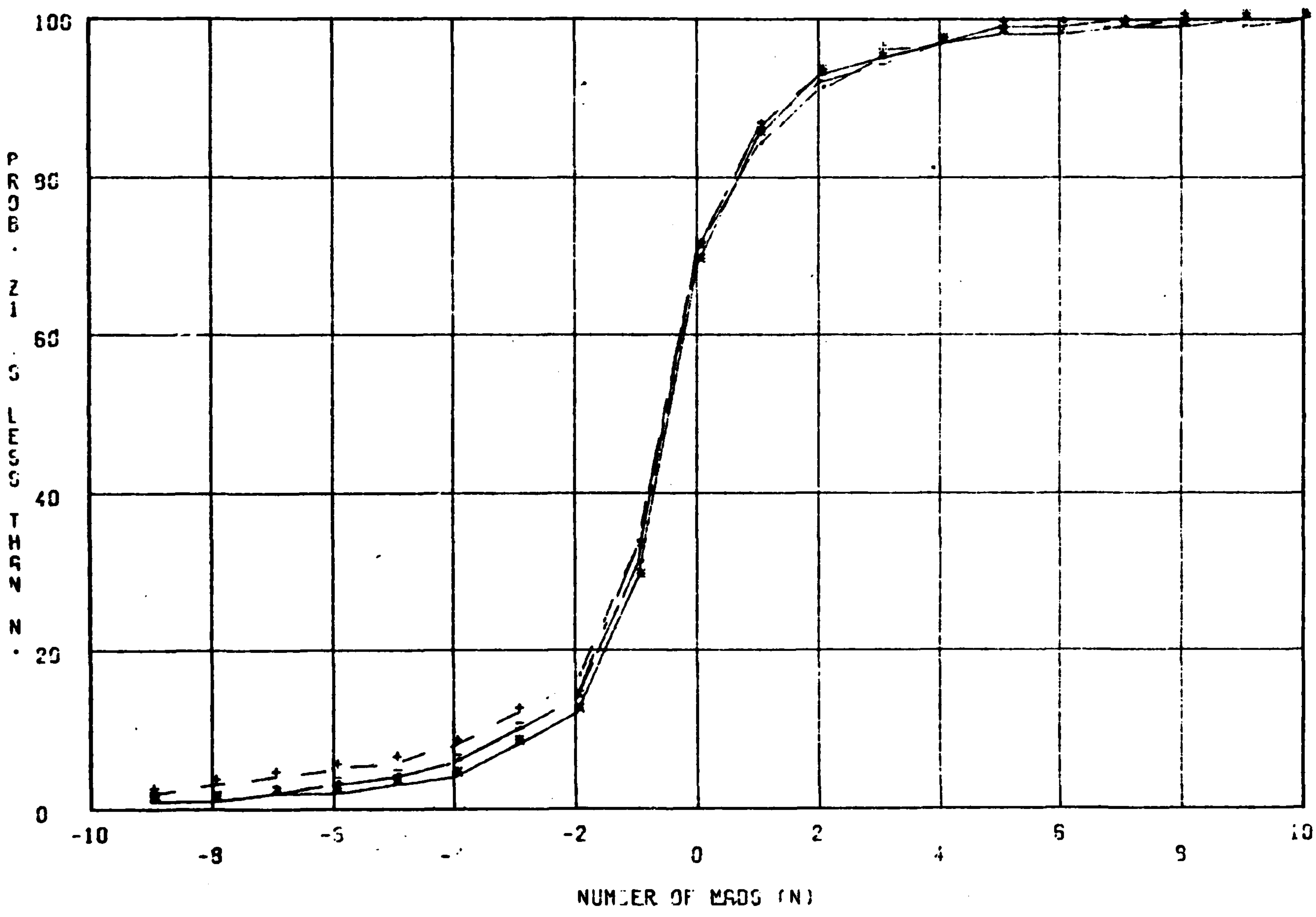
FOR SM.H

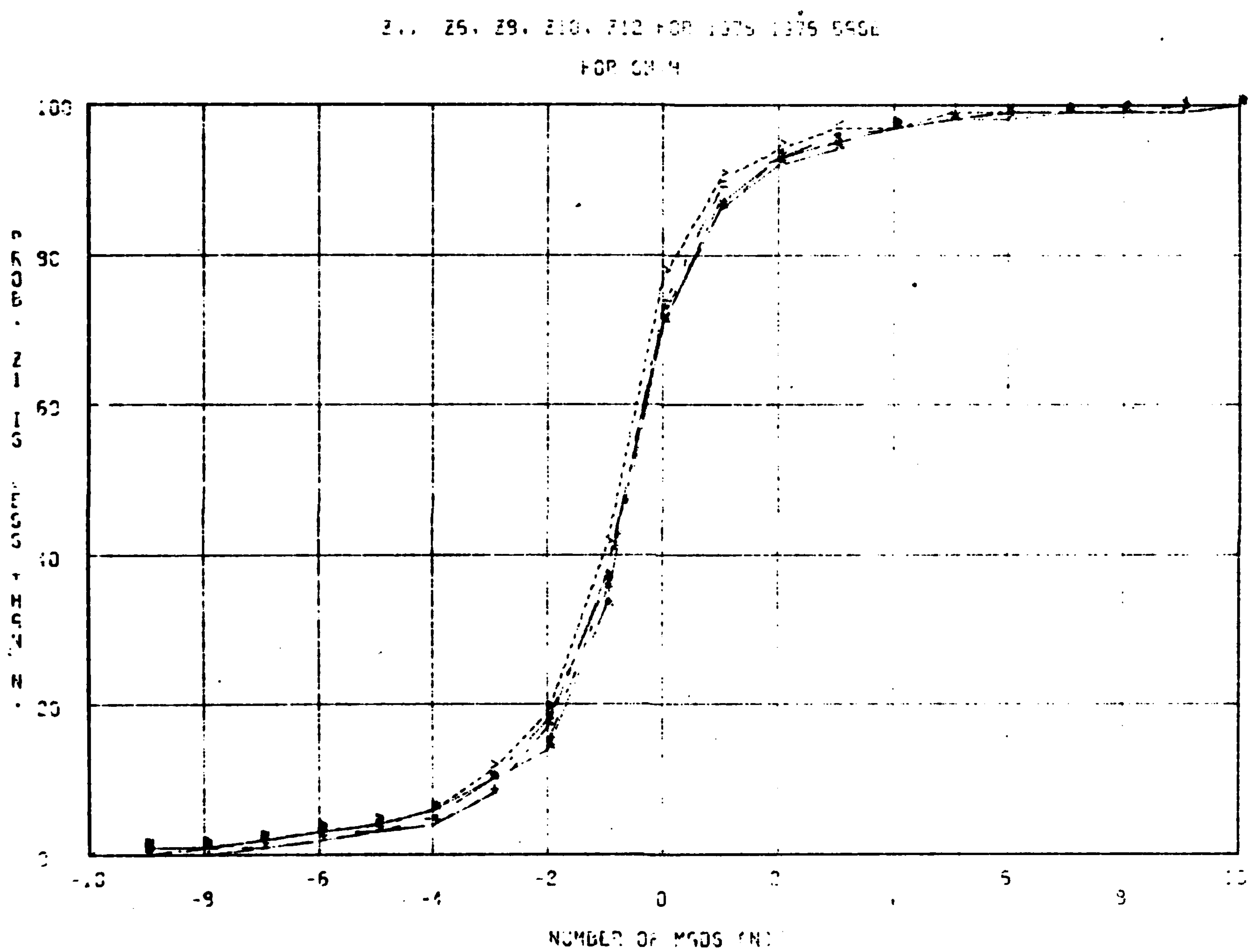


#5

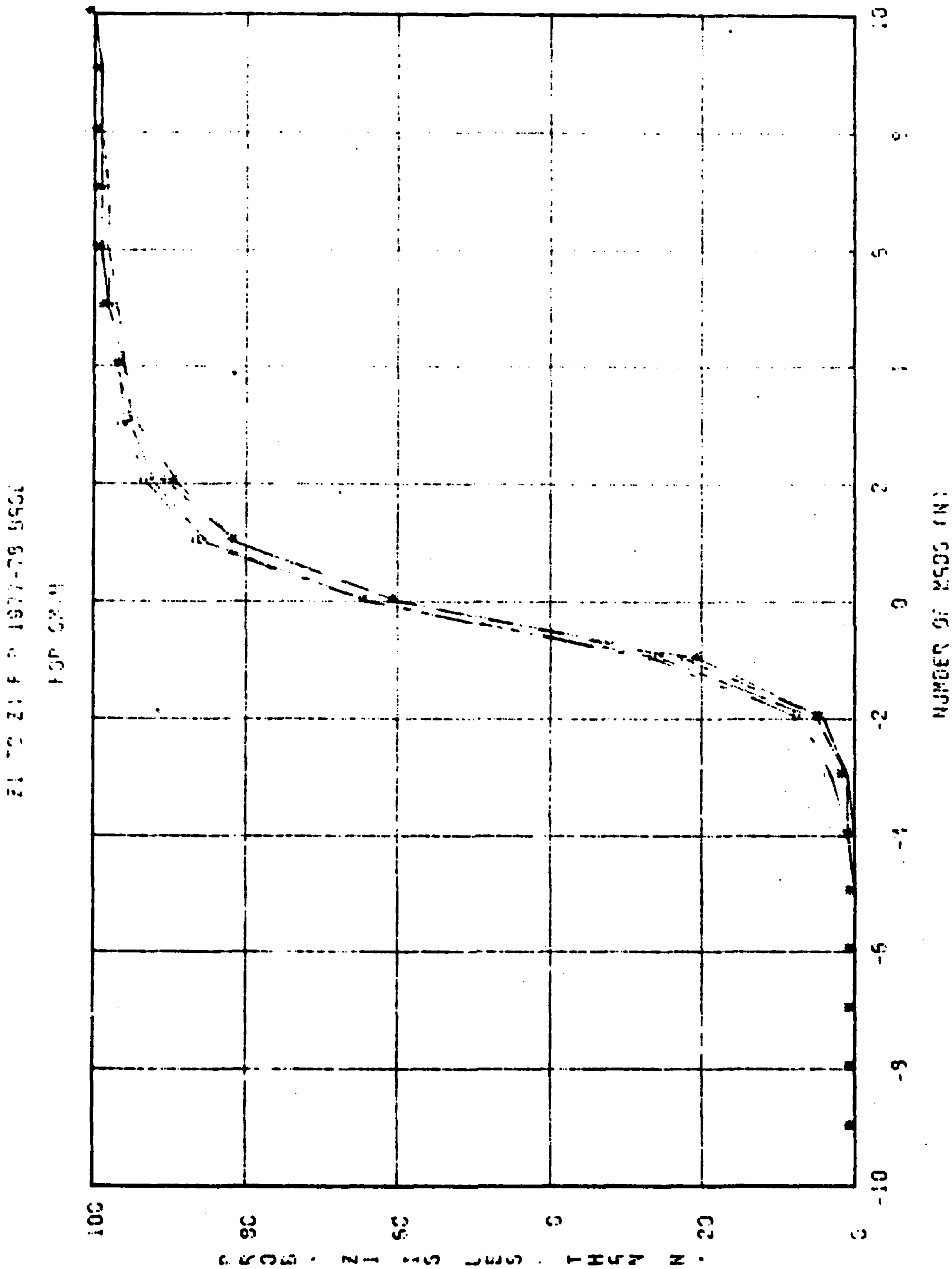
Z1 TO Z4 FOR 1975-76 BASE

FOR SM.4



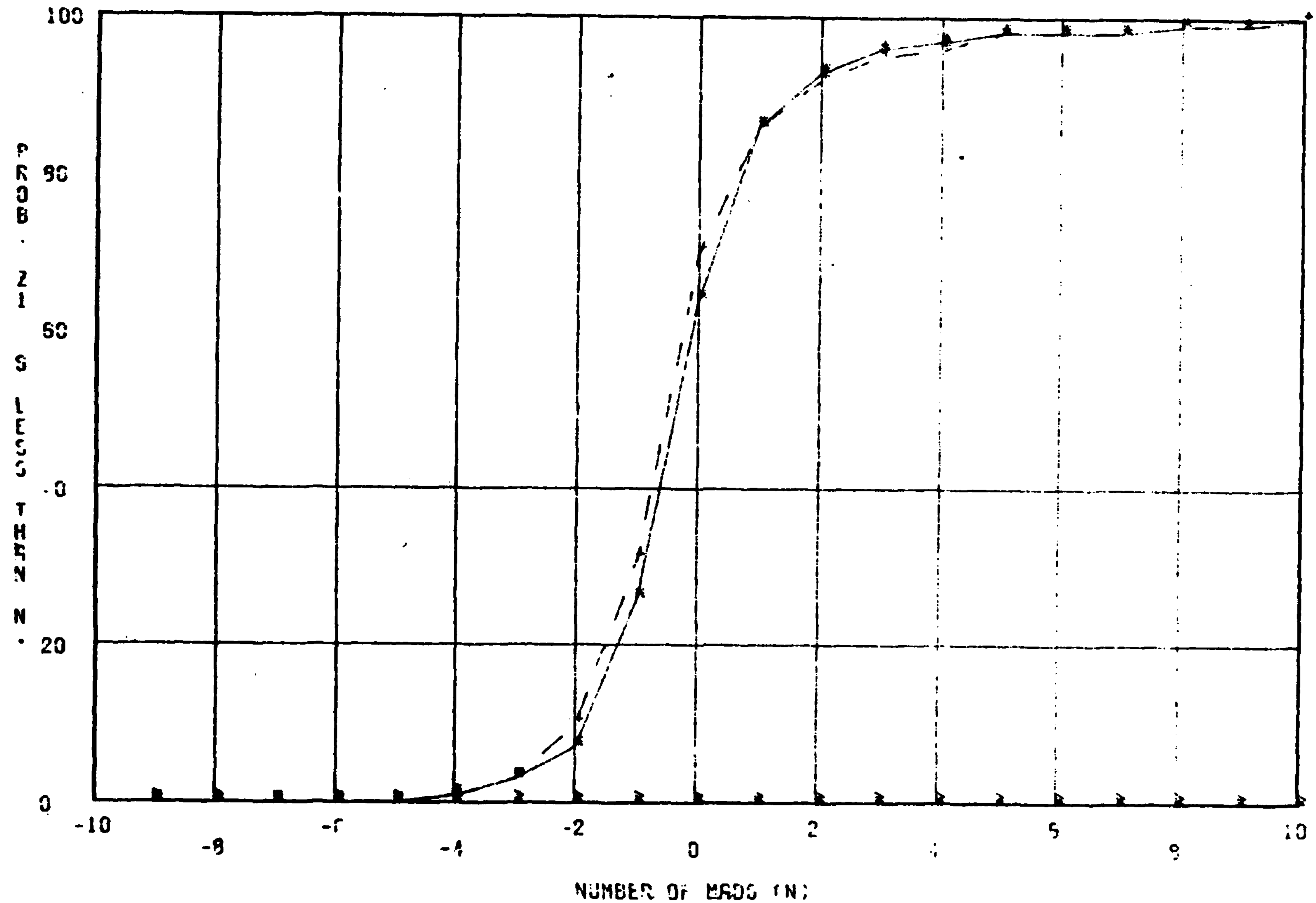


#6
H005
4-9-81
B-22



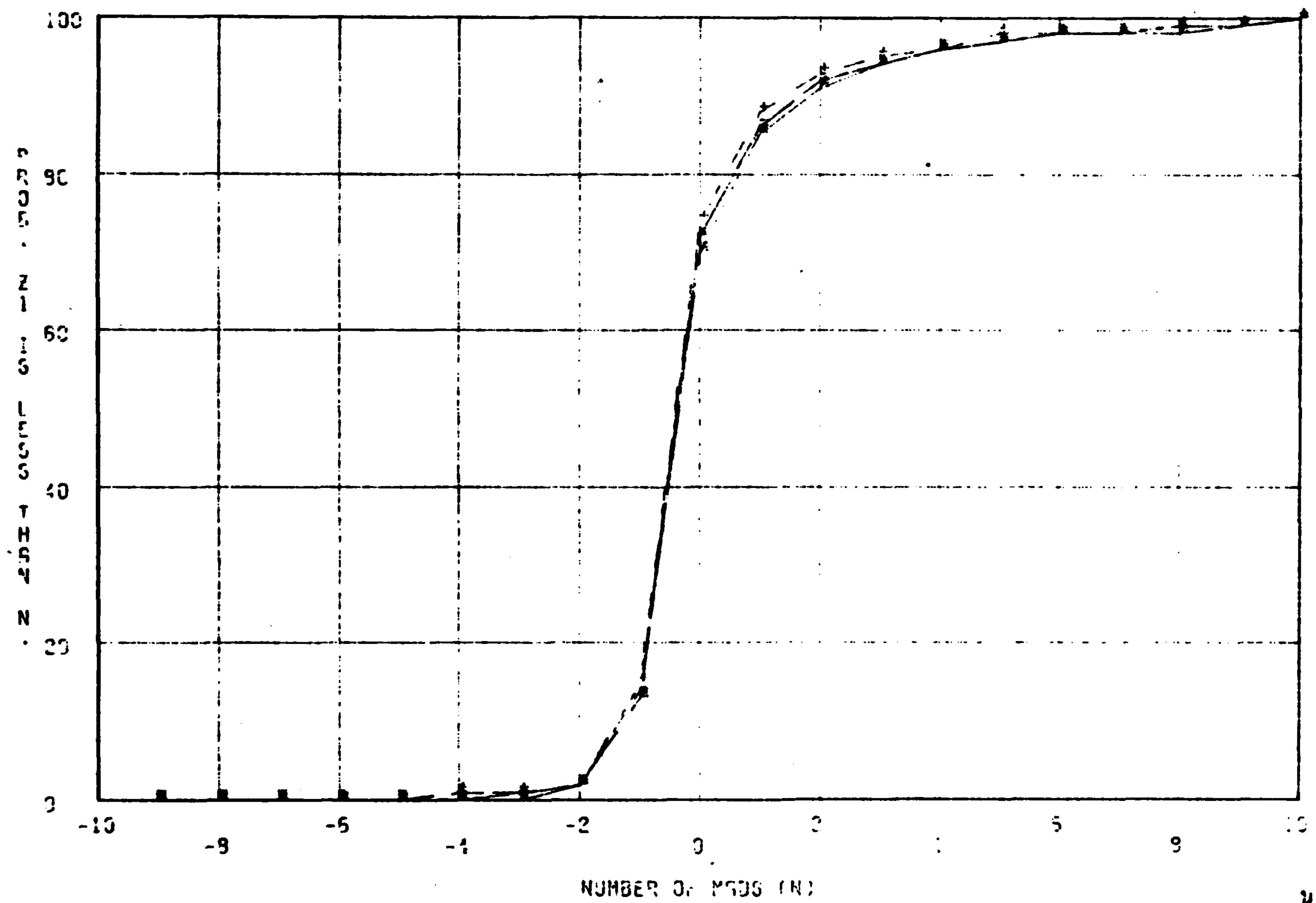
Z4, Z6, Z8, Z10, Z12 FOR 1977-1978 BSSE

FOR SM.4



Z1 TO Z1 FOR 1971-1972 BASE

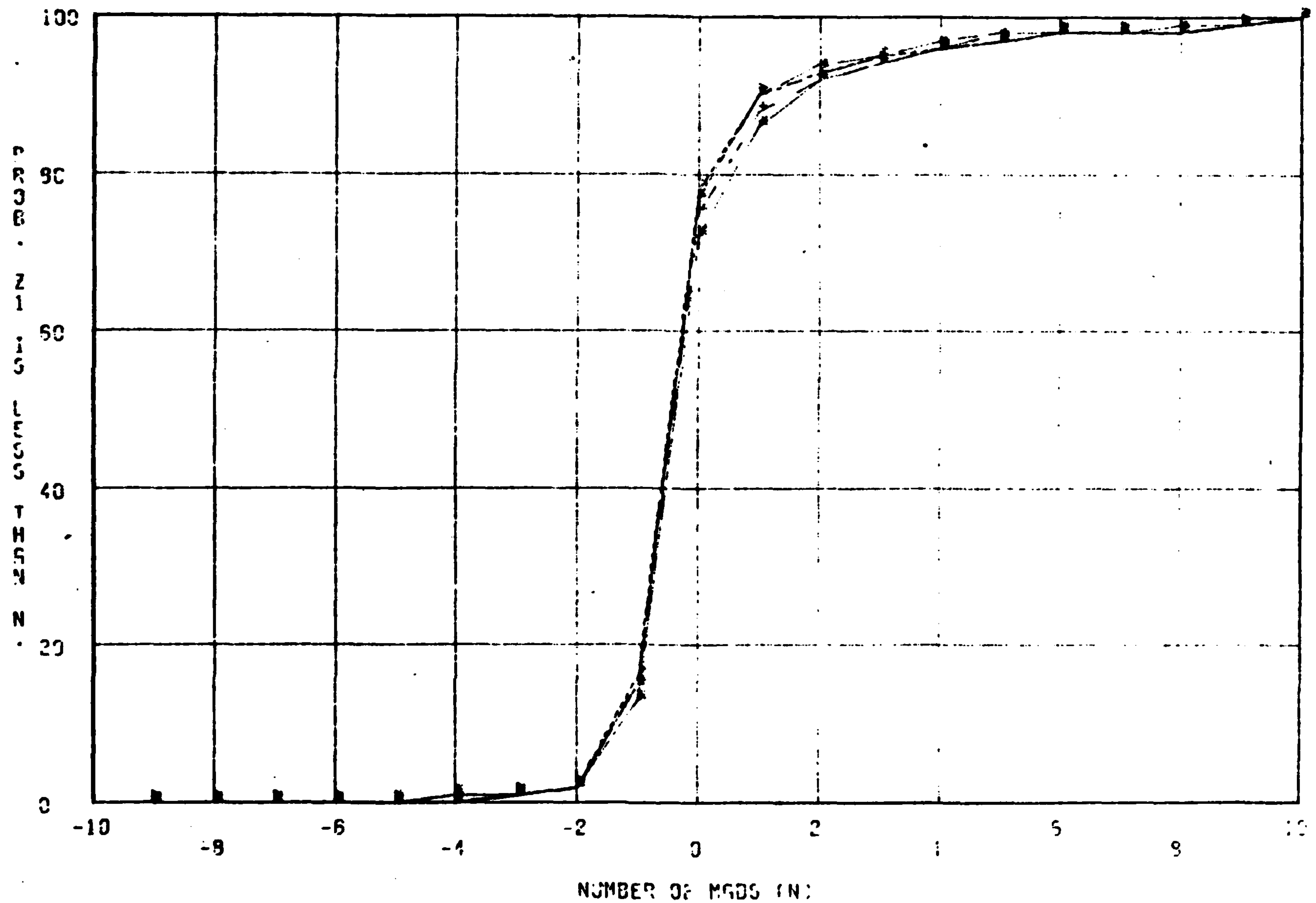
FOR [illegible]



#1
HCC7
4-9-81

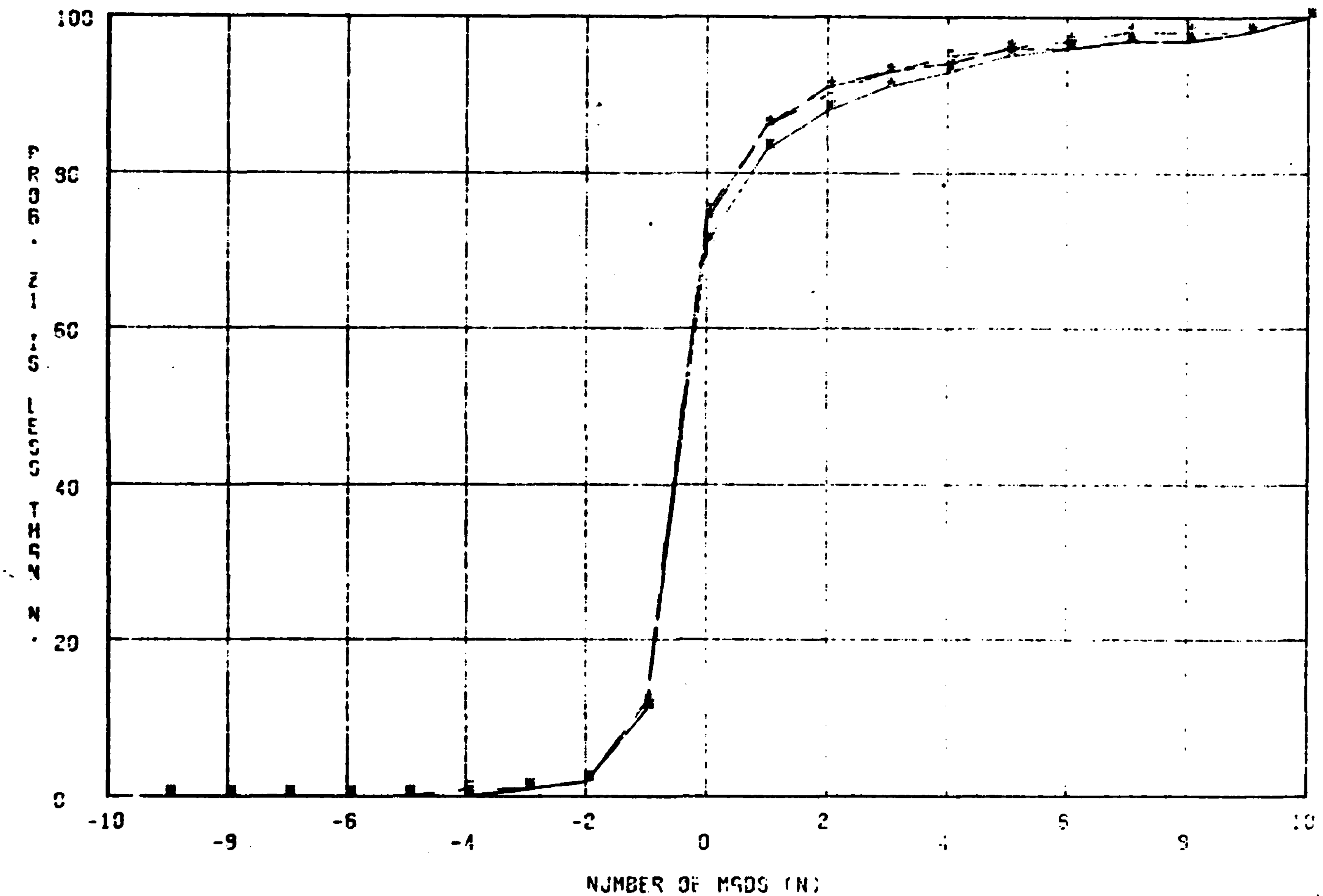
Z1, Z6, Z9, Z10, Z12 FOR 1971-1972 SASE

FOR SM.I



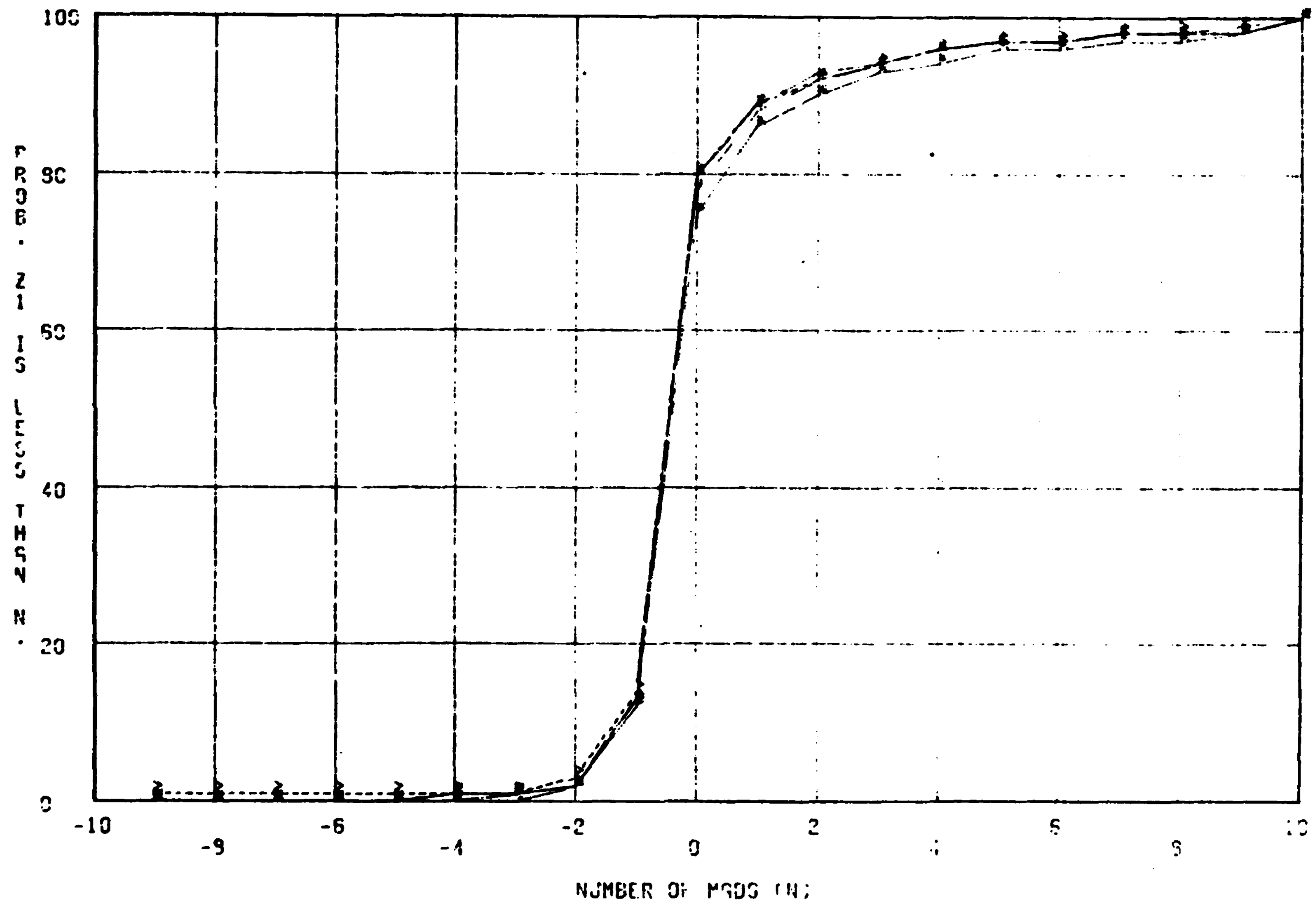
Z1 TO Z4 FOR 1973-74 DSSL

FOR SMAL



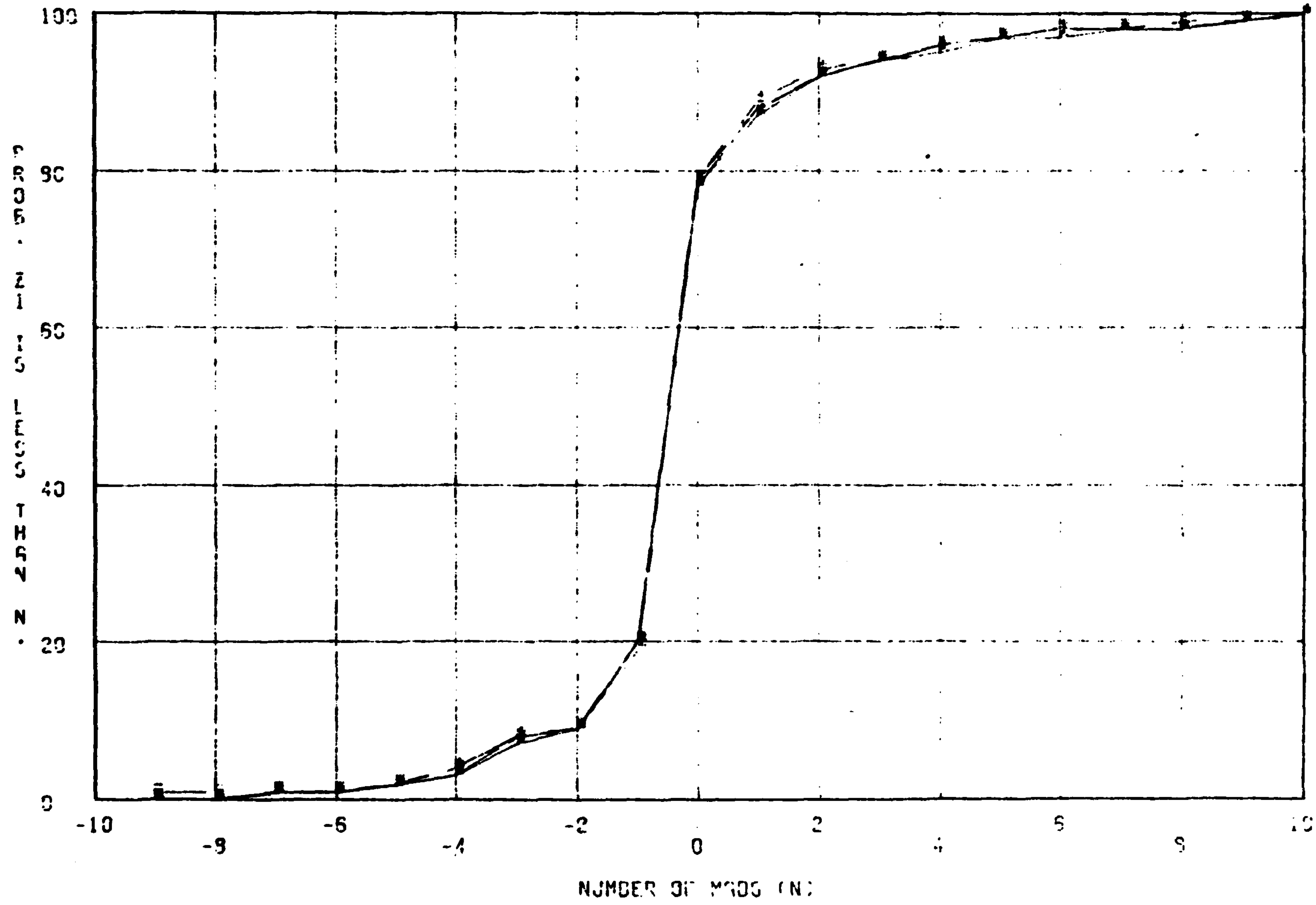
Z4, Z6, Z8, Z10, Z12 FOR 1973 1974 [illegible]

FOR SM.L



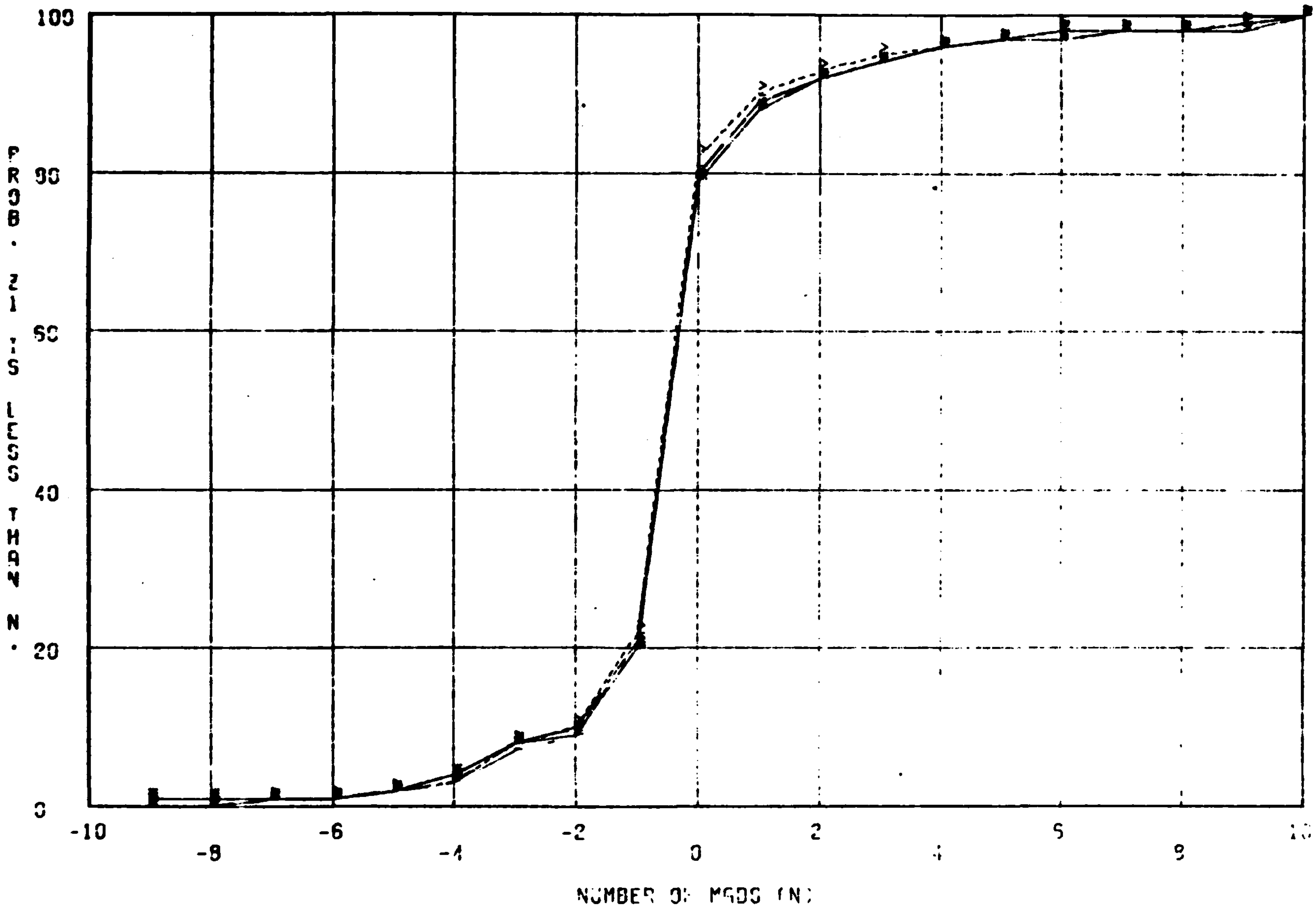
Z1 TO Z4 FOR 1975-76 NASL

FOR [illegible]



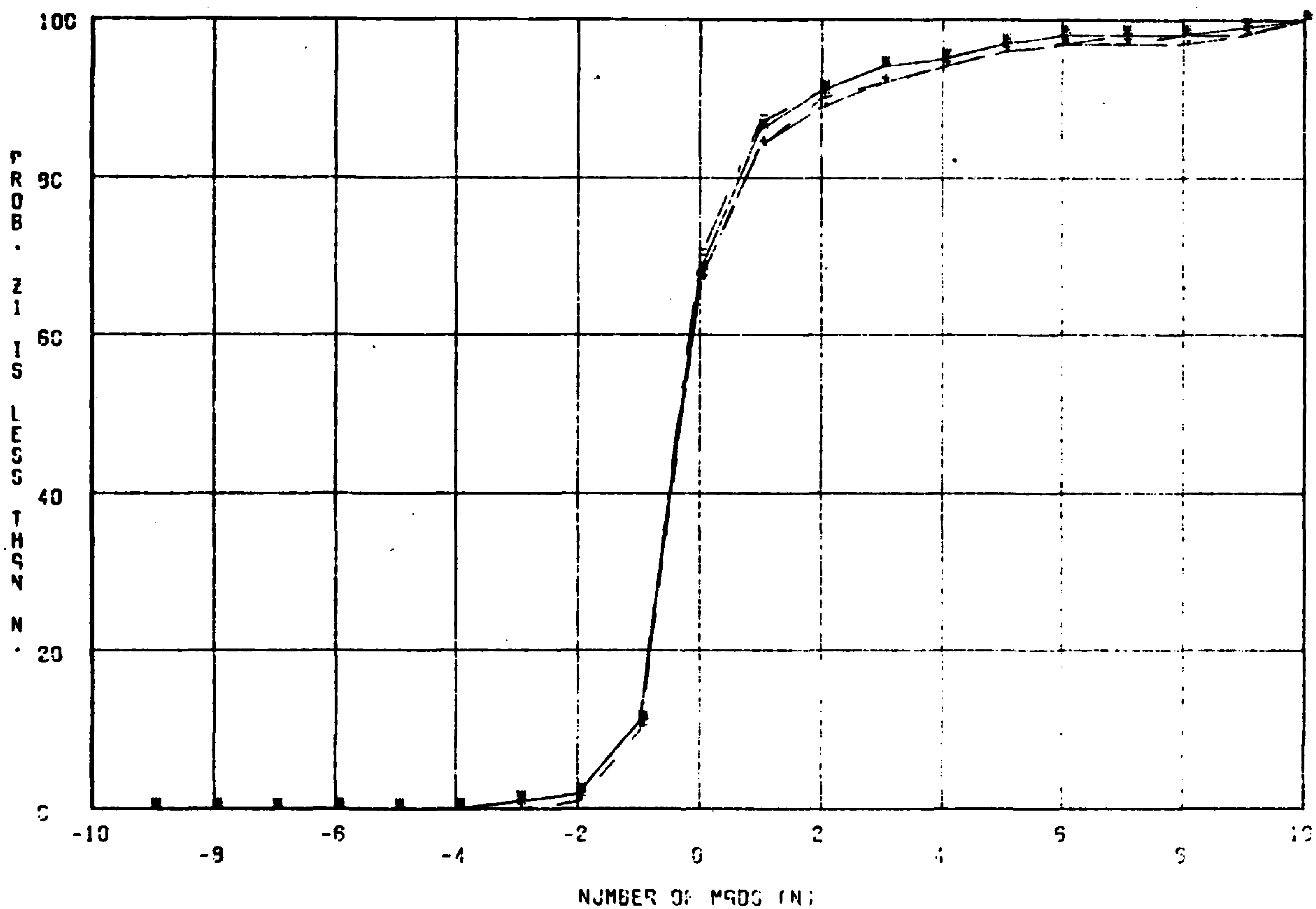
Z4. Z6. Z9. Z10. Z12 FOR 1975-1976 BASE

FOR SM.L



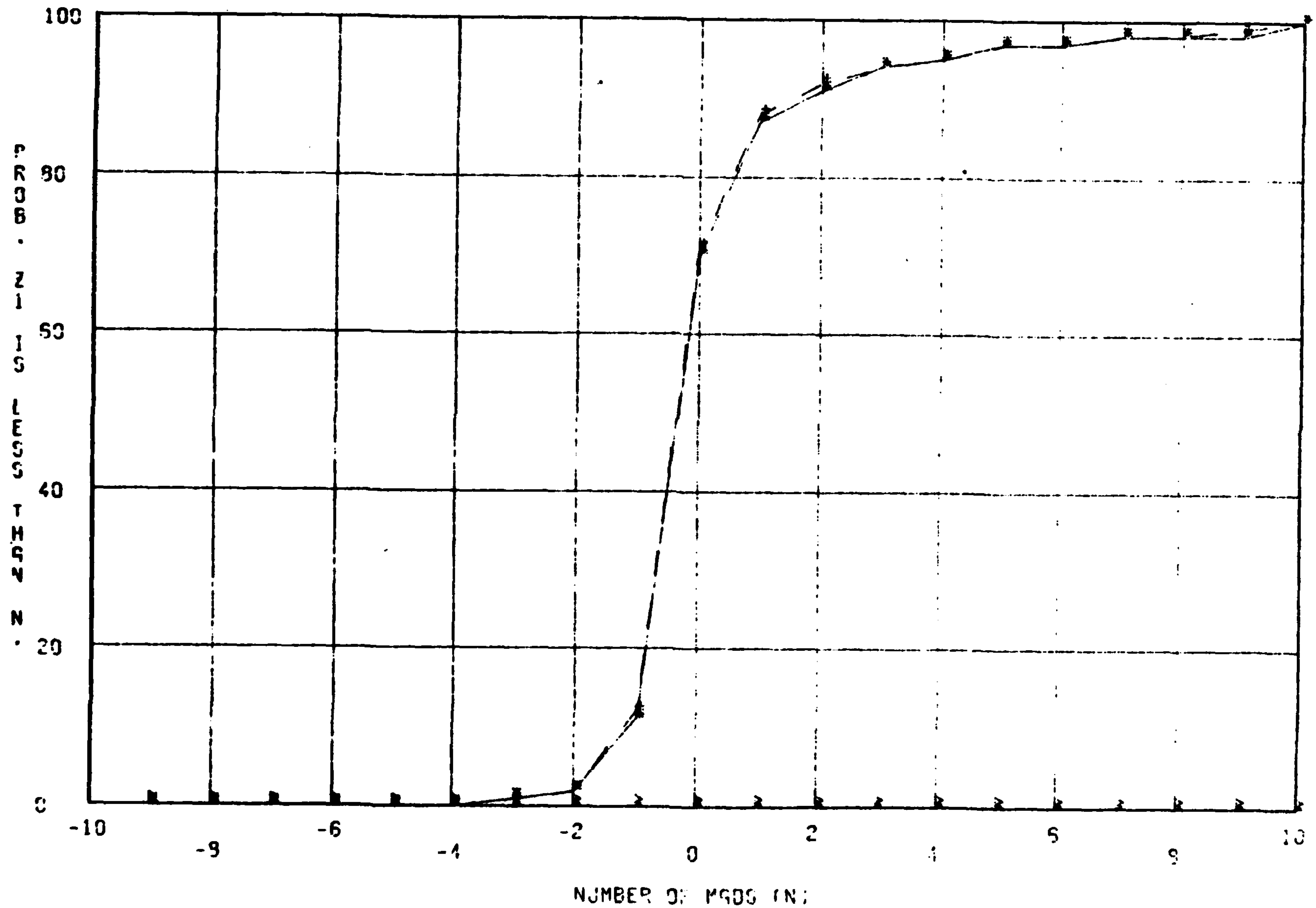
Z1 TO Z4 FOR 1977-79 BASL

FOR SM.L



Z4, Z6, Z8, Z10, Z12 FOR 1977-1978 BASE

FOR SM.1



Appendix C

Fortran Plot Program for Analysis of Low Demand Cummulative Distribution Functions

```
LIST 1-1000

100BM40PL.2
20C          PLOT CDF OF UNIT DEMAND FOR LOW MOVING ITEMS
30C
40       DIMENSION AVE(20),CDFNRM(11)
50       DIMENSION F(20),XX(100),YY(100)
60C
70       DIMENSION T(20,50),RMAD(20)
80       DIMENSION PTS(10),DASHCD(10)
90       CHARACTER OPTS*4(10),TITLE*50
100       CHARACTER ANS*1
110       CHARACTER FILEN*9
120C
130       DATA DASHCD/12.,32.,52.,7112.,7212.,7312.,
140&        7412.,7512.,3212.,5212./
150C
160C         NORMAL CDF VALUES FOR MAD= -5,-4,...,5
170C
180       DATA CDFNRM/0.,.1,.8,5.4, 21.1, 50.0, 78.8,
190&               94.5, 99.2, 99.9, 100.0/
200C
210C         INPUT DATA FILENAME
220C
230       PRINT ,"FILE-NAME;"
240       READ(5,13)FILEN
250C
260C         ADD A ";" TO FILEN FOR CALL TO ATTACH.
270C
280         ENCODE(FILEN,1)FILEN
290      1  FORMAT(A8,";")
300C
310         CALL ATTACH(07,FILEN,1,0,ISTAT)
320C
330C         INPUT CDF TABLES
340C
350       CALL FPARAM(1,132)
360C
370        PRINT,"PRINT INPUT TABLES? (Y OR N)"
380        READ(5,13) ANS
390C
400C
410       DO 49 JJ=1,48,12
420C
430        WRITE(6,3) FILEN,JJ,JJ+11
440     3   FORMAT(///,T30,A8,//T25,"TABLES ",I4," -",I4//)
450C
460       DO 39 I=1,20
470C
480         READ(7,13)LN,RMAD(I),(T(I,J),J=JJ,JJ+11)
490    13   FORMAT(V)
500C
510C         ADJUST RMAD
520       RMAD(I) = RMAD(I)-1
530         IF(ANS.EQ."Y") WRITE(6,23)LN,RMAD(I),(T(I,J),J=JJ,JJ+11)
540    23   FORMAT(I5,F4.0,12F4.0)
550C
560C         IF T(I,J) =0, RESET IT TO = .5
570C
580       DO 29 J=JJ,JJ+11
590        IF(T(I,J).LT. 0.5) T(I,J)=.5
600    29 CONTINUE
610C
620     39  CONTINUE
630     49  CONTINUE
640C
650       CALL UAIN(CHAR)
660C
670C
680C          DO PLOTS FOR TABLES 1-48, 6 CURVES AT A TIME
690C
700C
710       DO 890 JJ=1,48,6
720C
730       CALL USTART
740C
750       CALL USET("DEVICE")
760       CALL USET("PERCENTUNITS")
770C
780C
790       CALL USET("GRIDAXES")
800C
810C
820       DO 490 NPLT=1,1
830       GO TO (210,220,230),NPLT
840   210 CALL UDAREA(5.,45.,20.,90.)
850       ICOL=1
860       PTSX=20.
870       CALL USET('LINYAXIS')
880        GO TO 290
890C
900   220 CALL UDAREA(55.,95.,20.,90.)
910       ICOL=1
920       PTSX=20.
930       CALL USET('LNYAXIS')
940       GO TO 290
950C
960   230  CONTINUE
970       CALL UAIN(CHAR)
980         CALL UEND
990         CALL USTART
1000         CALL UDAREA(5.,95.,20.,90.)
ready

*
```

```
LIST 1000-2000

1000       CALL UDAREA(5.,95.,20.,90.)
1010      CALL USET('LNYAXIS')
1020     ICOL=10
1030     PTSX=10.
1040C
1050     GO TO 290
1060C
1070  290 CONTINUE
1080C
1090C          SET X(I) FOR WHEN NO. OF REQS. = 1
1100C
1110     CALL USET('NEWSCALE')
1120     CALL UPSET('XLABEL','NET UNITS OF DEMAND (N)\')
1130     CALL UPSET('YLABEL','P ( NET DEMAND <  N   )\')
1140C
1150     CALL USET('XBOTHLABELS')
1160     CALL USET('YBOTHLABELS')
1170C
1180     CALL USET('LINE')
1190     CALL UPLOT1(RMAD(ICOL),T(ICOL,JJ),PTSX)
1200C
1210C          NOW PLOT LEAD TIMES = 2,3,...,6
1220C
1230     DO 380 J=2,6
1240C
1250C
1260     CALL USET('OLDSCALE')
1270     CALL USET('DASHLINE')
1280     CALL UPSET('SETDASH',DASHCD(J))
1290C
1300     CALL USET('NOXLABEL')
1310     CALL USET('NOYLABEL')
1320C
1330    JCOL = JJ + J -1
1340     CALL UPLOT1(RMAD(ICOL),T(ICOL,JCOL),PTSX)
1350 380 CONTINUE
1360C
1370C
1380 490 CONTINUE
1390     CALL UPRINT(15.,15.,'OBSERVED NET UNIT DEMAND CURVES
1400     ENCODE(TITLE,493)FILEN,JJ,JJ+5
1410  493  FORMAT('FOR FILE = ',A8,
1420&         '    BASE DEMAND : ',I3, ' -',I3,'\')
1430      CALL UPRINT(35.,6.,TITLE)
1440C
1450C          RETURN TO ALPHANUMERIC MODE
1460C
1470C          COMPUTE AVERAGE VALUE FOR CURVE
1480C
1490     DO 539 I=1,20
1500      AVE(I)=0.
1510      DO 519 J=JJ,JJ+5
1520       AVE(I)=AVE(I)+T(I,J)
1530   519 CONTINUE
1540      AVE(I)=AVE(I)/6.
1550   539 CONTINUE
1560C
1570C          PLOT THE AVERAGE CURVE
1580C
1590     CALL UDAREA(55.,95.,20.,90.)
1600C
1610     CALL USET('NEWSCALE')
1620     CALL UPSET('XLABEL','NET UNITS OF DEMAND (N)\')
1630     CALL UPSET('YLABEL','P ( NET DEMAND <  N   )\')
1640C
1650     CALL USET('XBOTHLABELS')
1660     CALL USET('YBOTHLABELS')
1670C
1680     CALL USET('LINE')
1690     CALL UPLOT1(RMAD,AVE,20.)
1700C
1710C         SPIKE POISSON MODEL WITH EXTRA
1720C          MASS AT X=0.
1730C
1740C          ESTIMATE THE MEAN QTR DEMAND OF POISSON
1750C
1760     RMEAN = (JJ + (JJ+5))/8.
1770C
1780C
1790C
1800     P0=EXP(-RMEAN)
1810     F1=AVE(11)/100.
1820     A= (F1-P0)/(1.-P0)
1830     IF(A.LT.0.) A= 0.
1840     C = A
1850C
1860C         COMPUTE CUMMULATIVE SPIKE PCISSON PROBABILITIES
1870C
1880C
1890     XX(1) = 0.
1900     F(1) = AVE (11)
1910     P = (1.-A)*P0
1920C
1930     DO 530 I=1,9
1940C
1950C          UPDATE POISSON PROB
1960C
1970     P=P*RMEAN/FLOAT(I)
1980      XX(I+1) = I
1990      F(I+1) = F (I) +P*100.
2000   530 CONTINUE
ready
*
```

```
IST 2000-
      530 CONTINUE
C
C           NOT PLOT IT
C
C
          CALL USET('OLDSCALE')
C             COMPUT CUMMULATIVE SPIKE POISSON PROB
          CALL USET('DASHLINE')
          CALL UPSET('SETDASH',DASHCD(3))
          CALL USET('NOXLABEL')
          CALL USET('NOYLABEL')
           CALL UPLOT1(XX,F,10.)
C
          CALL UAIN(CHAR)
C-------------------------------------------------
C
C             EXPONENTIAL TAIL CALCULATIONS
C
C             FIT AND EXPONENTIAL TAIL THROUGH THE
C         AVERAGE CURVE POINTS FOR DEMAND = 0. AND 9.
C         SEE USD 7/2/81 NOTES FOR THE FORMULAS.
C
          F1=AVE(11)/100.
          Z1=RMAD(11)
          F2=AVE(19)/100.
          Z2=RMAD(19)
C
C             ESTIMATE COEF A AND B OF '1-F(Z) = A*EXP( B*Z )'
C
          B= ALOG(  (1. -F1)/(1.-F2) ) / (Z1 - Z2)
C
          A = (1. -F1)*EXP( -B*Z1)
C
C
C             EVALUATE F(Z) = 1 - A*EXP(BZ) FOR MAD'S ) 0
C
          Z=0.
          DO 579 I=1,20
          F(I) = 100.*(1.-A*EXP(B*Z) )
          XX(I) = Z
             Z = Z + 0.5
C
      579 CONTINUE
C
C             PLOT THE EXPONENTIAL APPROXIMATION
C
          CALL USET('DASHLINE')
          CALL UPSET('SETDASH',DASHCD(2))
          CALL UPLOT1(XX,F,20.)
C
C
C           PUT ON LABELS
C
          CALL UPRINT(60.,15.,'AVERAGE, POISSON AND EXPONENTIAL FITS\')
C
          ENCODE(TITLE,623)A,B,C
      623 FORMAT('A=',F8.3,'   B=',F8.3,'  C=',F8.3,'\')
C
          CALL UPRINT(60.,12.,TITLE)
C
          CALL UALPHA
C
          CALL UAIN(CHAR)
C
          CALL UEND
C
      590   CONTINUE
            WRITE(6,13)'END-OF-RUN.  STOP.'
          STOP
          END
ready
*
```

END

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